cordance with this fact, three possible values of λ, and to each a corresponding quadric linearly determined.

A known invariant identity has yielded readily the determining equation for the irrational invariant λ. I intend at another time to show that the reverse process is legitimate, direct and fruitful.

NORTHWESTERN UNIVERSITY,
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TWO BOOKS ON ELEMENTARY GEOMETRY.

Elements of Geometry. By ANDREW W. PHILLIPS, PH.D., and IRVING FISHER, PH.D., Professors in Yale University. New York, Harper and Bros. 1896. 540 pp. $1.75.

Elementary Solid Geometry and Mensuration. By HENRY DALLAS THOMPSON, D.SC., PH.D., Professor of Mathematics in Princeton University. New York and London, The Macmillan Company. 1896. vii+199 pp. $1.25.

Among the multitude of books on elementary geometry brought out within the last few years, two that have appeared recently rise above the general level and call for notice.

The larger and more ambitious of these—" Elements of Geometry," by Phillips and Fisher, a work of over 500 pages—covers the ground of the ordinary school text-books on plane and solid geometry, and makes the rough places smooth before the pupil with all the modern equipments in the way of realistic diagrams and conspicuous and varied types. We are tempted to wonder whether by thus diminishing the necessity for the exercise of the imagination and the attention, we may not lose some of the qualities which render geometry invaluable as a mental discipline. A noteworthy innovation is the final chapter, headed an "Introduction to Modern Geometry," which gives a brief account—so brief as to be scarcely more than a summary—of inversion, linkages, stereographic projection, poles and polars, the nine-points circle, perspective, duality, involution, antiparallels and the axioms of plane, spherical and pseudospherical geometries.

The chief novelties to be noted in the first part of the book are the early introduction of the idea of symmetry and the use of rotation about a point to prove theorems, relating to parallel lines. The note on p. 23, designed apparently to teach the beginner how far he may put his trust in axioms, shows a fine disregard of philosophic doubts. It says: "Lobatchewsky in 1829 proved that we can never get rid of the parallel axiom without assuming the space in which we live to be very different from what we know it to be through experience. Lobatchewsky tried to imagine a different sort of universe in which the parallel axiom would not be true. This imaginary kind of space is called *non-Euclidean* space, whereas the space in which we really live is called *Euclidean*, because Euclid (about 300 B. C.) first wrote a systematic geometry of our space."

The treatment of limits is clear and accurate and the definitions and proofs are, for the most part, well-worded and concise. The use of a strange assortment of small letters—generally m, s, w—to denote sometimes angles, sometimes lines, is objectionable; and such formulæ as those for the area of a triangle and the radius of its circumcircle in terms of the sides should hardly, if given at all, be placed among the examples.

On turning to the "Geometry of Space" the diagrams delight the eye. Each theorem is illustrated by a photogravure of a model constructed for the purpose of demonstrating it, and side by side with each of the photogravures is a simplified drawing showing merely the lines necessary to the demonstration. The book will help us to wait for that millennium when every teacher of solid geometry shall have models ready to his hand.

As regards the subject matter of this division the most obvious criticism is that there is too much of it; 120 theorems are collected and many of these, those, for instance, on radially situated polyedra, on spherical polygons and on the construction of the dodecaedron and icosaedron might well be spared, or relegated to the oblivion of an appendix. There is a lack of the gradation in difficulty that is desirable in a text-book for young pupils, while the more mature student should be led on to principles rather than encouraged to dally over isolated theorems. In many cases generalization has been carried too far and the most important facts, *e. g.*, the expression for the volume of a sphere, stated merely as corollaries.

The third section, on Modern Geometry, is too curtailed, it is to be hoped that in a future edition it may be expanded; remarks on space of constant curvature are of little value before curvature itself has been defined. The indices are useful and the book is well printed. The absence of any mention of conic sections is unexplained.

In direct opposition to the intuitional method of Drs. Phillips and Fisher is the logical form of Dr. Thompson's "Elementary Solid Geometry and Mensuration;" to quote from the preface, at the risk of being suspected of having read no further, "the attempt has been made in every case to secure the belief in the truth of the proposition from previous geometrical knowledge, rather than from any direct generalizations from material bodies." This attempt has been carried out in an able manner, and the result is a mathematical treatise sustaining a continued argument, instead of the arbitrarily arranged collection of theorems too often met with. The theorems given, though simple and few in number, are well chosen and sufficient to serve as an introduction to the higher branches of mathematics.

The straight line is frequently a source of difficulty to writers on elementary geometry. Many who have steered clear of the Scylla of a definition have fallen into the Charybdis of a postulate which fails to state the very property subsequently assumed as fundamental.

Dr. Thompson's postulate is as follows:

"There is in space a line, called the straight line, possessing the following properties:

(a) Any segment can be moved forward or backward in the line without flexing it.

(b) Through any two points there is one, and but one, such line." To which, he says, *may* be added "(c) A segment of it is a shortest line in space between *any* two points."

Disregarding the ambiguously worded shortest distance property a circle through a fixed point satisfies all the requirements formulated in (a) and (b), and yet a page or two further on (Cor. 3, § 18) we find the statement that "any straight line of the one plane may be brought into coincidence with any straight line of the other."

Other instances of similar slackness are, however, hard to find; and such theorems as "if two planes meet, they meet in at least two points" and "a pair of perpendiculars to the axis of a dihedral, one in each of the two faces, through a point on the axis, makes an angle which is the same, no matter where the point is taken on the axis," which are too often left to intuition, are here rigorously proved. The systematic use of capitals for points, small letters for lines and Greek letters for planes is a detail, but admirable, and the book throughout shows a scholarly treatment of an elementary subject.

ISABEL MADDISON.

BRYN MAWR COLLEGE, *February*, 1897.