accommodations and hospitality which the Society had enjoyed, and the meeting was adjourned.

THOMAS S. FISKE.

COLUMBIA COLLEGE, *September* 10, 1895.

THE MEETING OF THE AMERICAN ASSOCIATION.

THE forty-fourth annual meeting of the American Association for the Advancement of Science was held at Springfield, Mass., August 29–September 4, President E. W. Morley, of Cleveland, Ohio, presiding. The attendance was not large, only 367 members being registered. This is 110 less than the number attending the Brooklyn meeting last year. There were 113 new members and 58 fellows elected this year.

Excursions were given to Forest Park, Springfield; to Amherst, Northampton, and Mount Holyoke College, and to Holyoke.

The officers of Section A were as follows: Edgar Frisby, Washington, D. C., Vice-President; Asaph Hall, Jr., Ann Arbor, Mich., Secretary; C. A. Waldo, Greencastle, Ind., Councilor; J. E. Kershner, Lancaster City, Pa.; C. A. Doolittle, Philadelphia, Pa.; L. L. Conant, Worcester, Mass.; C. H. Rockwell, Tarrytown, N. Y., together with the Vice-President and Secretary, sectional committee; J. R. Eastman, Washington, D. C.; S. C. Chandler, Cambridge, Mass.; J. M. Van Vleck, Middletown, Conn., with the Vice-President and Secretary, committee to nominate officers of the section. A. N. Skinner, D. C., Press Secretary.

The work of the Section was devoted almost wholly to astronomy, only two papers in pure mathematics being presented. The tendency of the Section to devote itself to the former class of work was commented on and deplored, and it was hoped that this tendency might be arrested before it had gone too far, and the balance between mathematical and astronomical work restored at the next meeting. This feeling manifested itself to a certain extent in the choice of officers, for the ensuing year, mentioned below.

The papers presented to Section A were as follows:

1. *Development of some useful quaternion expressions, with applications to geometry of three and four dimensions.* By JAMES BYRNIE SHAW, Jacksonville, Ill.

2. *The constant of aberration.* By C. L. DOOLITTLE, Philadelphia, Pa.

3. *On the constant of nutation.* By S. C. CHANDLER, Cambridge, Mass.

4. *Progress of the zone-work at the Naval Observatory, Washington, D. C.* By A. N. SKINNER, Washington, D. C.
5. *Sun-spots and magnetic storms.* By M. A. VEEDER, Lyons, N. Y.
6. *Notes on square numbers whose sum is either a square or the sum of other squares.* By ARTEMAS MARTIN, Washington, D. C. Read by the Secretary.
7. *A convenient formula for computing times of moon rising.* By EDGAR FRISBY, Washington, D. C.
8. *On a slide scale for computing precession.* By EDGAR FRISBY, Washington, D. C.
9. *Some results for stellar parallax from meridian transit observations at the Washburn Observatory.* By A. S. FLINT, Madison, Wis.
10. *The spectrum of β Lyræ.* By E. B. FROST, Hanover, N. H. Read by the Secretary.
11. *Chronology and ancient eclipses.* By S. W. BALCH, Yonkers, N. Y.
12. *Period of R Comæ.* By H. M. PARKHURST, Brooklyn, N. Y.
13. *On the distribution and the secular variation of terrestrial magnetism.* By L. A. BAUER, Washington, D. C. By title.
14. *On making astronomy popular.* By Miss MARY PROCTER. By title.

No. 1 was a development of the A-process in quaternions, where A is defined by the equation $A\,.\,pq = \frac{1}{2}(pq - qp)$. This paper developed the four alternating functions

$$A\,.\,pq = \tfrac{1}{2}(pq - qp),$$
$$S\,.\,pAqr,$$
$$A\,.\,pqr = S\,.\,pAqr - Sp\,.\,A\,.\,qr - Sq\,.\,Arp - Sr\,.\,Apq,$$
$$S\,.\,pAqrs.$$

A set of four quaternions related to one another is deduced, analogous to a set of three rectangular unit-vectors. Various collections of formulas are deduced.

Affixing one of the set of four quaternions to each vertex of a tetrahedron, and letting $P = x_1 l_1 + x_2 l_2 + x_3 l_3 + x_4 l_4$ be that point for which the volumes $PABC$, $PBCD$, $PCDA$, $PDAB$ are as $x_1 : x_2 : x_3 : x_4$, we are enabled to treat solid geometry projectively.

No. 2 was a description of the method employed by Professor Doolittle in determining the constant of aberration. For this purpose a series of 2796 zenith telescope observations were made during the interval from Oct. 10, 1892, to Dec. 27, 1893. Of these observations, 1744 were made before midnight, and 1052 after midnight. Each observation gives an equation of the form

$$\phi_0 + k + Ax + By + Cz + Du + E\rho + \tau\mu = \phi$$

where x, y, z, u depend upon the two terms of the latitude variation, ρ is the correction to the assumed value of the constant of aberration, and μ the secular change of the latitude. These equations were combined so as to eliminate ϕ_0 and k, leaving 6 unknown quantities. A least-square solution gives for the constant of aberation $20''.552 \pm .009$.

No. 3 consisted of (*a*) a new determination of the constant of lunar nutation from an extensive body of observations, not hitherto employed for this purpose, and calculated by its distribution to avoid constant sources of error affecting many previous determinations; (*b*) a combination of this value with those deduced by previous investigations, to determine the most probable value of this fundamental constant of astronomy.

No. 4 detailed the relation of that part of the work at the Naval Observatory at Washington under Mr. Skinner's personal direction, to the general scheme of the zone-work planned by the German Astronomical Society. A description was given of the methods employed and the instruments used in this work, and mention was made of the difficulties encountered and of the methods of overcoming them as the work progressed.

No. 5 exhibited printed tables, not heretofore published, illustrating methods of recording sun-spots and magnetic phenomena so as to bring out their relations to each other, and the periodicities involved. Dr. Veeder believed that the records of magnetic observatories might be put in form so as to show the extent of the daily disturbances in such a manner that they could be printed and made readily accessible to the public.

No. 6 developed theorems relating to

(1) n square numbers whose sum is a square.

(2) Three square numbers the sum of whose squares is a square.

(3) Five square numbers the sum of whose squares is a square.

(4) n square numbers the sum of whose squares is a square.

(5) Three square numbers whose sum is equal to the sum of two other squares.

(6) Four square numbers whose sum is equal to the sum of two other squares.

(7) Four square numbers whose sum is equal to the sum of three other squares.

(8) m square numbers whose sum is equal to the sum of n other squares.

(9) $S(1^2) + S(2^2) + S(3^2) + S(4^2) + \ldots\ldots + S(n^2) = \square$, where $S(x^2) = 1^2 + 2^2 + 3^2 + \ldots\ldots + x^2$.

(10) Three square numbers, the sum of every two of which is a square.

(11) Four square numbers, the sum of every three of which is a square.

This paper will be published in the *Mathematical Magazine.*

No. 9 detailed the method of observation used at the Washburn Observatory, which is the same as that employed at the Leyden Observatory, 1885–87, by Prof. J. C. Kapteyn. It consists in observing the differences in the time of meridian transit between the parallax star and two companion stars, one preceding and one following. A list of 76 stars was observed, the Repsold meridian circle being used. These observations included all stars known to have a proper motion of 1″ or more; and some rapid binaries and stars of the first and second magnitnde were added later. The results obtained agree with those obtained by previous observers. This paper is to be published in the *Astronomical Journal.*

No. 10 gave a brief account of some of the problems regarding the spectrum of β Lyræ, which have been brought to light by the recent spectroscopic observations of this star at Pulkowa, Potsdam, and London.

No. 11 contained some original researches relating to the bearing on astronomical computation of certain irregularities in the Roman calendar. In astronomical calculations which extend back to early periods of history, it is customary to follow the Old Style calendar. When astronomical events thus computed are also dated in the historical record it is found that the record and computation agree as to the day of the month, provided the date is subsequent to March 1 of 1 B.C. Prior to that date the Roman calendar was more or less irregular, but it has not been customary to follow these irregularities in astronomical work. Instead, the Old Style calendar has been extended backward as far as necessary for calculation; and is, to that extent, fictitious. The actual calendar as observed by the Romans differs from the fictitious or astronomical calendar by amounts varying according to the time. It is important in historical comparisons to know what this difference is; and this was shown in tables prepared for the purpose by Mr. Balch. History indicates that Cæsar began the calendar with a new moon. Computation shows that in 46 B.C. the moon was new on January 2 of the Old Style calendar. Mr. Balch's table showed that the kalends of January as observed coincided with the day preceding January 1 of 46 B.C. The new moon occurred, therefore, two days after the calendar began.

Hence Cæsar's astronomer, Sosogenes, was so much out in his reckoning; but the people doubtless did not observe it, supposing that the new undertaking was begun with the new moon, in accordance with their superstitions.

No. 12 gave the results of observations from 1831 to 1895, on R Comæ, from which it is evident that the period of this star, which was formerly 361 days, has become 365 days; and that, if the period continues to lengthen, the maxima will soon cease to be observable. If, however, we adopt a sine formula, equally well representing all the observations, we find that the period will shorten, bringing the maximum periods out of the twilight and rendering them observable. But if the period should continue to lengthen, the maximum will not be visible again for many years. Either of these theories accords perfectly well with recorded observations; but the former, depending as it does on the sine formula, seems now the more probable.

The next annual meeting of the Association will be held at Buffalo. Instead of beginning in the middle of the week, as has been the custom during the past few years, the sessions will begin on Monday and continue until Friday night. No excursions are to be given until after the close of the meeting. The President for the ensuing year will be E. D. Cope, of Philadelphia.

The officers of Section A for 1896 are as follows: Vice-President, W. E. Story, of Worcester, Mass., and Secretary, E. B. Frost, Hanover, N. H. LEVI L. CONANT.

WORCESTER, MASS., *September* 14, 1895.

ON THE DIFFERENTIAL EQUATIONS OF CERTAIN SYSTEMS OF CONICS.*

BY MR. R. A. ROBERTS.

IN a few cases certain systems of curves can be represented very simply by differential equations of the first order. The latter, since they do not involve the parameter, give readily the tangents to curves of the system, at an arbitrary point of space or of the surface to which the curves are confined. Hence certain loci connected with the system and various properties of the curves can be obtained without having recourse to the integral. A remarkable example of this is to be found in the case of the geodesics on a quadric, all whose

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