

result by a much shorter and more satisfactory process. He must not fear that he is giving his mind to a subject too trivial. No matter how slight the addition which he makes to the sum of knowledge, it is yet an addition; and unless it is superseded by the doing of the same thing by some one else in a better manner, it is a permanent contribution to science. Some are helped greatly, at times, by working first on some numerical illustration of the problem in hand; others, again, by a preliminary geometrical representation; and the first path to any discovery is not usually the best. It is sometimes supposed that the mass of original work done in so many countries and published in so many languages makes it likely that any ordinary piece of work will be overlooked in the great mass. Nevertheless, *litera scripta manet*; and what may now seem an unimportant addition to an unimportant branch may probably one day, when that branch is no longer unimportant, and when its special history comes to be itself a topic of discussion, receive its due recognition. Meantime, every little helps. The most trifling addition to the actual sum of knowledge will be at least useful as a step to aid the next investigator; but whether important or unimportant, whether appreciative recognition comes or not, whether others are helped or no one takes notice, there is a degree of personal pleasure in the mere fact of origination which is the just and certain reward of every piece of successful investigation.

NOTE ON A MEMOIR IN SMITH'S COLLECTED PAPERS.

BY PROFESSOR CAYLEY.

AMONG the most noticeable papers in the Collected Mathematical Papers of H. J. S. Smith we have the hitherto unpublished "Memoir on the Theta and Omega Functions," XLIII (vol. 2, pp. 415-623), written in connection with Dr. Glaisher's Tables of the Theta Functions and originally intended as an Introduction thereto. It appears that in 1873 or 1874 Dr. Glaisher asked him, as a member of the British Association committee for the calculation of the Tables, whether he would contribute an Introduction. His reply was that he did not see his way to writing anything appropriate to the tables themselves, but that he "could say something with respect to the constants at the head of the pages." These constants were K, K', E, J, J' , etc., the numerical values whereof were given for every minute of the modular angle. The memoir grew in extent, and it was finally decided that it should follow these yet unpublished tables with the before-mentioned title,

“Memoir on the Theta and Omega Functions,” but fortunately it has at length appeared in the Collected Papers as above.

In explanation of the title and scope of the memoir, it will be remembered that the Theta Functions are functions of two arguments, x and q ; so that giving to x the value zero or any numerical value, or any value depending on that of q , we obtain a series of functions containing the single argument q , or writing as usual $q = e^{i\pi\omega}$, say the single argument ω ; and in the memoir the attention is directed chiefly but not exclusively to these functions of a single argument which are termed *Omega Functions*. The functions chiefly considered under this designation are Hermite's functions $\phi\omega$, $\psi\omega$, $\chi\omega$, which represent the values of \sqrt{k} , $\sqrt{k'}$, and $\sqrt{k k'}$ considered as functions of $q = e^{i\pi\omega}$. To fix the ideas, it may be mentioned that the actual values (in one of their very numerous forms) are

$$\phi\omega = \sqrt{2}q^{\frac{1}{4}} \prod_1^{\infty} \frac{1 + q^{2m}}{1 + q^{2m-1}},$$

$$\psi\omega = \prod_1^{\infty} \frac{1 - q^{2m-1}}{1 + q^{2m-1}},$$

$$\chi\omega = \sqrt{2}q^{\frac{1}{4}} \prod_1^{\infty} \frac{1}{1 + q^{2m-1}},$$

each of them a one-valued function of ω : any rational and integral function of $\phi\omega$, $\psi\omega$, $\chi\omega$ is termed a *Modular Function*. It is right to add that the definitions extend only to those values of q for which the series are convergent, or (what is the same thing) ω regarded as a point must be situated within a certain region of the upper infinite half-plane $y = +$.

The requisite formulæ for the Theta Functions are obtained from Jacobi's fundamental formula for the multiplication of four Theta Functions; and the Elliptic Functions are introduced by means of their definitions in terms of the Theta Functions: and the whole theory of Elliptic Functions is thus brought into connection with the Theta and Omega Functions. The theory of Transformation depends in a great measure on the arithmetical and geometrical theory of binary matrices, of which the constituents are integral numbers; this theory plays an extensive part throughout the memoir.

An abstract of the contents of the memoir is as follows:

- Arts. 1-14. Definitions and Elementary Properties of the Theta, Omega, and Elliptic Functions.
 15-23. Arithmetical Theory of Binary Matrices.
 24-34. The Transformation of the Theta and Omega Functions.
 35-45. Geometrical Representation of Binary Quadratic Forms.
 46-51. Geometrical Representation of the Modular Functions $\Phi\omega$ and $\Psi\omega (= \phi^s\omega, \psi^s\omega)$.
 52-58. The Modular Equation.
 59-62. The Equation of the Multiplier.
 63-73. The Modular Curves.
 74-82. Theory of the Modular Functions $\phi\omega$ and $\psi\omega$.
 83-88. Theory of the Modular Function $T\omega = (1 - \chi^{2^s}\omega)^2 \div \chi^{2^s}\omega$.
 89-90. The Differential Equation of the Modular Equations and Curves (this last section somewhat incomplete).

A good deal of the same ground is gone over in Weber's *Elliptische Functionen und Algebraische Zahlen* (8vo, Brunswick, 1891), a work which exhibits in a very compendious form the higher parts of the theory of Elliptic Functions, and which well deserves to be carefully studied.

NOTES.

THE Annual Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York on Friday afternoon, December 28, at three o'clock. There were seventeen members present. In the absence of the president and vice-president, Professor R. S. Woodward occupied the chair. Reports were presented by the secretary and treasurer. The secretary stated that the membership of the Society was 251. The average attendance at the ordinary meetings during the year had been 16, the attendance at the last annual meeting 24, and that at the summer meeting 22. The number of members who had attended at least one meeting was 60. An auditing committee was appointed to examine the treasurer's accounts.

The annual election being then in order, the chair appointed Dr. Blake and Dr. Stabler tellers. Upon examining the ballots cast in person and by mail, they announced that the following ticket had been elected :