tion of laws; still, *a priori*, the hypothesis would seem to be quite admissible. Certain details of the theory remain to be fixed as may be most advantageous. For instance, the medium may have either one or more dimensions besides the familiar three, and its extent in the new directions is as yet undetermined. It may be self-returning in the new directions, instead of ending abruptly; and so on. It may even *shade off* in the new directions, the materials of the world tending toward our three-fold space as a region of maximum density; either through some kind of selection such as Mr. C. S. Peirce has suggested, or of quasi-attraction, a little as in Hinton's *Scientific Romances*. Such shading would follow some rapid exponential law, as in the theory of distribution of errors; and its modulus, however small, would probably be definite.

The useful assumption "that the measure of distance remains the same everywhere" (p. 201), does not necessarily imply that every so-called "curved space" lies in an uncurved space of more dimensions, but only that the relation among the mutual distances of four quasi-complanar points be always that known to hold among the geodetic distances apart of four points on a sphere whose radius is either real or purely imaginary. Thus it is not true that a two-dimensioned pseudospherical space must be finite unless "constructed in space of four dimensions" (pp. 200–1): and the oversight, made by so good a writer, goes to justify Klein's criticism that the phrase "curved space" is misleading. Why not rather, in the general case, describe the space as *quasi-curved?*

The rest of the account given of hyperspace and of the constitution of matter is well thought out and clear. It will help the general reader toward some truthful notions as to studies which may perhaps play an important part in the near future.

J. E. OLIVER.

ITHACA, N. Y., *November* 15, 1892.

RECENT STAR CATALOGUES.

Zweites Münchener Sternverzeichniss, enthaltend die mittleren Oerter von 13200 *Sternen, für das Aequinoctium*, 1880. Beobachtet und berechnet von Dr. JULIUS BAUSCHINGER, Observator der Sternwarte. Munich, 1891. 4to, pp. xxvi. and 172.

THE Scottish astronomer, John Lamont, long director of the observatory at Bogenhausen, a suburb of Munich, had caused the observation, about the middle of this century, of about 33,000 stars; they were taken in zones, and by an ingenious method for saving labor. Unfortunately, the assist-

ants, who made these observations, had fallen into a good many errors, and a thorough revision was necessary. This was begun by Feldkirchner, then the principal assistant at the observatory; and the corrected catalogue was published some years ago as vol. I. of the *Neue Annalen*, by Prof. Seeliger, Lamont's successor. In this revision it was found necessary to reobserve about 13,000 stars, those, namely, of the 33,000 which were not sufficiently controlled by duplicate observations and other catalogues. All these were observed, usually once, but occasionally twice or more, by Dr. Bauschinger, helped in reading the microscopes by Mr. List. The instrument was the old Reichenbach circle of the observatory, with an aperture of 11 centimetres; it has been provided with microscopic reading in place of Reichenbach's verniers.

The observations were accomplished between 1884 and 1888; stars were added to employ the time where large gaps existed in Lamont's zones; and the results have been published in the form of a catalogue for the epoch 1880. Many of the 13,000 stars are faint ones, picked up by Lamont's assistants on especially clear nights, and therefore less likely to be duplicated at Bogenhausen or elsewhere; while their faintness rendered it difficult for Bauschinger to see them on average nights. These especially difficult stars seem to be observed with probable errors of $\pm 0^s.14$ and $\pm 1''.3$ in right ascension and declination respectively. These numbers are not too large for the circumstances. Dr. Bauschinger estimates the probable error of a zone-star observed under average conditions at $\pm 0^s.081$ in right ascension, and $\pm 0''.85$ in declination; while a comparison made by myself, but including only stars to $8^m.2$ inclusive, gave probable errors about $\frac{1}{10}$ less in amount. The limit of magnitude corresponds nearly to that adopted by Argelander at Bonn, with an instrument a little larger, as the boundary of his classification of "brighter" stars, such as can be readily seen under average circumstances with full field illumination. The same limit, modified to suit an aperture of 12·2 centimetres, I have found to hold good for the Repsold circle at Williamstown; stars below $8^m.5$ require less field illumination than I habitually use unchanged for brighter ones. Dr. Bauschinger's work there is entirely adequate to its purpose. This would not, in fact, require this degree of accuracy; but it is also a source of star-positions, which are useful in many other ways. In a paper later in the same volume, Dr. Bauschinger discusses the proper motions of 90 stars. These are such as were detected in comparing Lamont's and other catalogues. After a proper motion was thus made highly probable other existing catalogues were searched for data, and, after bringing everything up to 1880, a least square discussion was undertaken with partial allowance for system-

atic corrections. The weights here assigned are rather a rude approximation ; this is pardonable where the object is mainly to make sure of the existence and general direction of the proper motion. But when the purpose in view is to obtain the most probable values of the motions, the systematic corrections and the weights should be rather more strictly applied.

Lalande's zones, for example, usually deserve no more than about $\frac{1}{20}$ of the weight of the Pulkova catalogue of 1855, while here we find $\frac{1}{4}$ employed instead. But this secondary correction is, as above stated, here of quite subordinate importance.

T. H. SAFFORD.

NOTES

A REGULAR meeting of the NEW YORK MATHEMATICAL SOCIETY was held Saturday afternoon, November 5, at half-past three o'clock, the president in the chair. The following persons, having been duly nominated, and being recommended by the council, were elected to membership : Professor Cleveland Abbe, U. S. Weather Bureau ; Professor Henry S. White, Northwestern University ; Mr. Gardner Ladd Plumley, Home Life Insurance Company of New York. It was announced that the president and secretary had been made members of the international committee on the proposed joint memorial to Gauss and Weber at Göttingen. No formal papers having been announced, a general discussion upon mathematical topics was in order. Miss Williams made some remarks, in which she stated that Steiner's method of proving, that the circle has a greater area than any other plane figure having an equal perimeter, appeared to lack rigor, because he took for granted that a maximum exists, and the latter proposition had not been demonstrated. Edler has found a rigorous proof of this proposition relating to the circle ; but for the corresponding proposition with reference to the sphere, no rigorous proof by elementary geometrical methods seems to be known. Steiner himself objects to a similar method of proof, which Lhuilier applies to triangles. After stating his objection, Steiner gives a concise and rigorous proof of this proposition. When he goes on to the circle, he seems to forget his objection and to adopt a somewhat similar method. Most of the discussion which followed Miss Williams's remarks turned on this point. Dr. Fiske made some general remarks upon a recent paper * treating of the mathematical theory of

* Mathematical Investigations in the Theory of Value and Prices, by Dr. Irving Fisher, Yale University. *Transactions of the Connecticut Academy*, vol. IX., July, 1892.