multiplied by the divergent series

$$1 + \tfrac{1}{2} + \tfrac{1}{3} + \ldots$$

gives an absolutely convergent product. The strangeness of this last conclusion is removed when we consider that the series

$$-1 + \frac{1}{1.2} + \frac{1}{2.3} + \ldots$$
$$= -1 + (1 - \tfrac{1}{2}) + (\tfrac{1}{2} - \tfrac{1}{3}) + \ldots = 0.$$

Since one of the factor-series is zero, we may well have a product-series with a definite limiting value. This value in this case is itself zero, as is seen from the following expression for the product-series

$$W = -c_1 + \sum_1^\infty{}_\nu (c_\nu - c_{\nu+1}), \text{ where } c = \sum_1^\nu{}_x \frac{1}{x(\nu + 1 - x)}.$$

COLORADO COLLEGE, *March* 23, 1892.

ON EXACT ANALYSIS AS THE BASIS OF LANGUAGE.*

BY A. MACFARLANE, SC.D., LL.D.

Abstract.

A SCHEME for an artificial language was published in the Philosophical Transactions of the Royal Society for 1668 by Bishop Wilkins. Since, however, it presupposes a complete enumeration of all that is or can be known, it would be overthrown by every considerable advance in knowledge. The mathematician and philosopher Leibnitz devoted much thought to what he called a *specieuse générale*, which he hoped would be an aid in reasoning and invention; but he died without publishing even an outline of his system. The new universal language Volapük, invented by J. M. Schleyer of Constance, is built upon a purely linguistic basis, being derived from a comparative study of the chief natural languages. In this paper it is proposed to show that the proper and necessary basis for an artificial language is scientific analysis and classification, and two specimens of language

* Abstract of a paper presented to the Society at the meeting of March 5, 1892.

so constructed will exhibit the great complexity of the problem.

In the notation for numbers in Volapük we observe serious defects. As regards the digits there is no word to express 0. As regards the expressions for the denominations, an arbitrary use of the affix for the plural denotes the denomination *ten*: thus we have *tel*, two; *tels*, twenty; and the other names for the denominations are no more systematic than the English words. There is the usual jump from thousand to million; we are not told whether *telion* means thousand million or million million; and no words are provided to express fractional denominations. In physical works we meet with the highest development of the notation for number; it consists of a series of significant figures, and of a positive or negative power of ten. To vocalize this notation we require an elementary word for each of the elementary numbers, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; and a series of words for the integer powers of ten, and for the fractional powers of ten. As there are five elementary vowels, ten words for the digits may be obtained by prefixing the consonants *b* and *l*.

Thus	0,	1,	2,	3,	4,	5,	6,	7,	8,	9,
	ba,	*be,*	*bi,*	*bo,*	*bu,*	*la,*	*le,*	*li,*	*lo,*	*lu.*

The word for a higher number is formed by taking the appropriate monosyllables in succession; for example: 11, *bebe;* 23, *bibo;* 105, *bebala.* The integer denominations may be expressed by affixing *p* to the number for the place or power of ten, while the fractional denominations may be expressed by adding *n* instead of *p*, thus:—

$10,\ 10^2,\ 10^3,\ 10^4,\ 10^5,\ 10^6,\ 10^7,\ 10^8,$ etc.
bep, bip, bop, bup, lap, lep, lip, lop, etc. and

$\frac{1}{10},\ \frac{1}{10^2},\ \frac{1}{10^3},\ \frac{1}{10^4},\ \frac{1}{10^5},\ \frac{1}{10^6},\ \frac{1}{10^7}$ etc.
ben, bin, bon, bun, lan, len, lin, etc.

For example, one hundred and twenty-three thousand would be vocalized by *bebipo bop*, and forty-five hundredths by *bula bin.*

Some years ago in a series of papers on "An analysis of the relationships of consanguinity and affinity," * the author devised a system of notation both literal and graphical, and indicated a corresponding nomenclature. On this analysis may be constructed another specimen of a scientific language, and by the system of words it provides for such relationships the efficiency of Volapük may be tested.

* *Proc. Roy. Soc. Edinb.*, Vol. X., p. 224; Vol. XI., pp. 5 and 162; Phil. Mag. June 1881; and Journal of the Anthrop. Inst. of London for 1882.

Let *a* denote the relationship of parent and *e* the reciprocal relationship of child; by forming the different permutations of these letters we get expressions for the several compound relationships. Those of the second order are :—

NOTATION.	GENERAL MEANING.	IRREDUCIBLE MEANING.
aa	parent of parent.	grandparent.
ae	parent of child.	consort.
ea	child of parent.	brother or sister.
ee	child of child.	grandchild.

The meaning given in the third column may not coincide exactly with that given in second; where a reduction of the expression is possible, that is, where *a* is followed by *e* or *e* by *a*; the special or reduced meaning is excluded. Thus *ae* and *ea* each in its most general meaning includes *self*; when the special meaning of *self* is excluded, the *parent of child* becomes *consort*, and the *child of parent* becomes *brother* or *sister*.

Similarly the relationships of the third order are :

NOTATION.	GENERAL MEANING.	IRREDUCIBLE MEANING.
aaa	great grandparent.	great grandparent.
aae	grandparent of child.	parent-in-law.
aea	parent of child of parent.	step-parent.
aee	parent of grandchild.	child-in-law.
eaa	child of grandparent.	uncle or aunt.
eae	child of parent of child.	step-child.
eea	grandchild of parent.	nephew or niece.
eee	great grandchild.	great grandchild.

In the case of all these relationships, excepting the first and the last, the general meaning includes a simpler relationship to which it may reduce; for example, grandparent of child includes the simpler relationship of parent. In the same manner the relationships expressed by four, five or any number of elements may be exhibited.

To change this notation into a nomenclature, all that is necessary is to insert some consonant as *d* between the vowels; for then each combination can be easily pronounced. In the systematic language so derived *ada* means grandparent, *ade* consort, *eda* brother or sister, *ede* grandchild, *adada* great grandparent, *adade* parent-in-law, *adeda* step-parent, and so on.

Each genus of relationship is divided into species by introducing the distinction of sex. Let the consonants *m* and *f* denote male and female respectively, then the species of the first order are *ma* father, *fa* mother, *me* son, *fe* daughter. If we introduce the distinction of sex after the vowel we obtain such relationships as *mam* father of man, *maf* father of woman, *mef* son of woman. The species of the second order, obtained by introducing the distinction of sex before the first vowel only, are, *e.g.*; *mada* grandfather, *feda* sister, *fede* granddaughter. If the distinction of sex is introduced before the second vowel also, we may obtain : *mama* paternal grandfather, *mame* father of son, *fema* sister-german, *fefe* daughter of daughter, etc. Thirty-two species may be formed by introducing the distinction of sex after the last vowel, but four of these species reduce necessarily to the relationship of *self*; for example *mamem*. The double relationship involved in full brother may be denoted by *memfa*, that of full sister by *femfa*, and that of full brother or sister by *emfa*. If, on the other hand, we wish to express that the brothership is only half, we may replace *d* by *t*; thus *meta*, half-brother; *feta*, half-sister; and *eta*, half brother or sister. These principles suffice to supply a word for every possible relationship of consanguinity or affinity. The nomenclature is based on a notation which serves as the basis for a calculus,* and it seems to me that this is a developed specimen of the kind of language which Leibnitz had in his thoughts.

If we test Volapük by the vocabulary which it provides for these relationships we find that the words supplied are not founded on a scientific analysis, and indeed are far inferior to the terms supplied by the English language. Almost all the stem words, as *son*, son, *blod* brother, involve the masculine gender, the corresponding feminines being formed by prefixing *ji*. Thus daughter is expressed by *ji-son* and sister by *ji-blod*. There are no words to express the relationships which are independent of sex. The confusion on the subject of the more involved relationships is very great, no distinction being made for example, between step-brother and half-brother, both of which are denoted by *lafa-blod*. The derived relationships are not expressed by general rules for combining the elementary relationships, but on the contrary a few words are obtained in an arbitrary manner by attaching to the stems comparatively meaningless prefixes and affixes. It has been pointed out by several scholars,† that the inventor of Vola-

* Problems in Relationship, *Proc. Roy. Soc. Edinb.*, 1888.

† Dr. D. G. Brinton—"Aims and Traits of a World-language;" Dr. Horatio Hale—"An International Language," *Proc. A. A. A. S.*, Vol. XXXVII.

pük makes a fundamental error in proceeding synthetically instead of analytically, and in this matter of terms for relationship we have an example of that fundamental mistake.

NOTES.

A REGULAR meeting of the NEW YORK MATHEMATICAL SOCIETY was held Saturday afternoon, April 2, at half-past three o'clock, the president in the chair. The following persons having been duly nominated, and being recommended by the council, were elected to membership: Mr. B. S. Annis, Johns Hopkins University; Professor Samuel Marx Barton, Emory and Henry College; Dr. Maxime Bôcher, Harvard University; Mr. William H. Butts, Pontiac, Michigan; Dr. T. Proctor Hall, Clark University; Professor S. W. Hunton, Mount Allison University; Mr. W. F. King, Ottawa, Canada; Mr. B. M. Roszel, Johns Hopkins University; Dr. Arthur Schultze, New York. The proposed amendment to the Constitution (*Bulletin*, No. 6, p. 142.) was unanimously adopted, and the By-Laws were amended by striking out section 2 of by-law IX., and altering the number of the following section. The following original papers were read: "The cubic-projection and rotation of a tessaract," by Dr. T. Proctor Hall; "On final formulas for the algebraic solution of quartic equations," by Professor Mansfield Merriman.

A *tessaract* is a geometrical figure generated by the motion of a cube in the direction of the common perpendicular to its edges and faces, bearing exactly the same relation to a cube that a cube bears to a square. It is bounded by eight cubes, and has twenty-four faces, thirty-two edges, and sixteen vertices. Dr. Hall presented the Society with a wire model representing the projection of a tessaract into space of three dimensions.

THE Cambridge University Press has in preparation "A treatise on the mathematical theory of elasticity," by A. E. H. Love, fellow of St. John's College, Cambridge. The first volume of the work, which is to be in two volumes, is in press.

MACMILLAN & Co. have nearly ready a work on the "Theory of functions," by Professor Morley of Haverford College, Pa., and Professor Harkness of Bryn Mawr College, Pa.

AT the meeting of the *Académie des Sciences* at Paris on