determined how much each star was delayed in observation by this process ; using an ingenious method invented by Bessel of artificially diminishing the light of the stars as seen through the telescope without altering the character of the image, and so found that his own mental processes delay his judgment by about a hundredth of a second per magnitude ; that is, he would observe a star of the eighth magnitude seven-hundredths of a second later than one of the first in the same place ; and so put it forward a second of arc and a small fraction in right ascension.

On the other hand, the Albany observations of right ascension are rather better, one by one, than those made at Helsingfors. This was probably in part due to Krueger's anxiety about his declinations, which gave him more trouble, owing to the weakness of his instrument in that respect. Fearnley, on the other hand, had a zone so far north (65° to 70°) that with the old method he was able to equal the quality of Boss' work in right ascension with the new, while his employment of verniers instead of reading microscopes has somewhat impaired his declinations.

But, all told, the uniformity of the three catalogues, due to the excellent plan formulated by Argelander, is more sensible and far more important than the trifling discrepancies in execution. The plan is in fact the quintessence of modern practical astronomy in the subject with which it deals. That it has been so warmly welcomed and so thoroughly executed by astronomers over the whole civilized globe is at once a proof of the excellence of their training and of the great advance which has been made in giving the human mind control over its own processes and over material objects.

TRUMAN HENRY SAFFORD.

A PROBLEM IN LEAST SQUARES.

BY PROF. MANSFIELD MERRIMAN.

To determine, by the method of least squares, the most probable values of a and b in the formula $y = ax + b$ when the observed values of both y and x are liable to error.

I. LET x_1 and y_1, x_2 and y_2, ... x_n and y_n be n pairs of observed values of two variables known to be connected by the relation

$$y = ax + b.$$

If the observed values of x were free from error, the most probable values of a and b would be deduced by the application of the common rules of the method of least squares. There would then be n observation equations of the form

$$ax + b - y = 0,$$

from which would result two normal equations

$$[x^2]\, a + [x]\, b - [xy] = 0,$$
$$[x]\, a + nb - [y] = 0,$$

whose solution gives for a the value

$$a_1 = \frac{n\,[xy] - [x]\,[y]}{n\,[x^2] - [x]^2}.$$

II. If however the observed values of y are free from error the formula should be written

$$\frac{1}{a} y - \frac{b}{a} - x = 0\,;$$

then by forming the normal equations and solving, there is found for a the value

$$a_2 = \frac{n\,[y^2] - [y]^2}{n\,[xy] - [x]\,[y]},$$

which in general is quite different from that given in I.

III. How shall the most probable value of a be found when the observed values of both x and y are subject to error ? The following is the solution which I made in February, 1891, when considering the problem at the request of the Director of the Observatory of Harvard College :

Let the weight of each observed value of y be unity, and let the weight of each observed value of x be g. Then let a_1 and a_2 be computed by the formulas in I. and II. The most probable value of a is then one of the roots of the equation

$$a^2 + \left(\frac{g}{a_1} - a_2\right) a - g = 0.$$

IV. The demonstration of the last formula will be given in full in a paper which is to appear in the report of the U. S. Coast and Geodetic Survey for 1890. Here there is only space to illustrate its application by one or two numerical examples.

When a has been computed the most probable value of b is directly found from

$$b = \frac{[y] - a\,[x]}{n}.$$

V. An interesting corollary is applicable to the case where a is known *a priori* and a_1 and a_2 are derived from observations. Then from III. the value of g is

$$g = \frac{a - a_2}{\frac{1}{a} - \frac{1}{a_1}}$$

VI. As an example of the application of III. and IV. let the following be simultaneous observations of two thermometers having the same exposure :

No. :	1	2	3	4	5	6	7	8	9
y :	9°	10°	10°	11°	11°	11°	12°	12°	13°
x :	10°	10°	11°	10°	11°	12°	11°	12°	12°

It is required to find the relation between the scales, or the values of a and b in the formula $y = ax + b$, regarding the weights of the two series as equal.

Here $g = 1$, $n = 9$, $[y] = 99$, $[x] = 99$, $[x^2] = 1095$, $[y^2] = 1101$, $[xy] = 1095$. These inserted in I. give $a_1 = 1$ and inserted in II. give $a_2 = 2$. Then from III. there results :

$$a^2 + (1 - 2)\,a - 1 = 0.$$

from which $a = +\,1.618$ and $a = -\,0.618$. The former of these is the value required (since it makes the sum of the squares of the residual errors a minimum, the latter making that sum a maximum). From IV. the value of b is now found to be $-\,6.798$. Thus,

$$y = 1.618x - 6.798$$

is the most probable relation resulting from the given observations. The common rules of the method of least squares would give $y = x$ if observed values of x be taken without error and $y = 2\,x - 11$ if observed values of y be without error.

VII. As an illustration of the use of V. let the following be estimations of the magnitudes of stars by two observers :

No.	:	1	2	3	4	5	6
y	:	8°	9°	10°	10°	10°	11°
x	:	9°	9°	11°	9°	10°	9°

It is required to find the weight g, it being known *a priori* that $a = 1$. Here, from I. there is found $a_1 = \frac{22}{29}$, and from II. $a_2 = \frac{32}{22}$; then from V. there results

$$g = \frac{22 \cdot - 32}{22 - 29} = \frac{10}{7},$$

or the weight of the first series of observations is to that of the second as 7 is to 10.

VIII. If the equation between the variables be of a degree higher than the first, as $z^2 = aw^3 + b$, values of a and b may be deduced by following the above method, regarding z^2 and w^3 as observed values corresponding to y and x. Since, however, the real observed values are z and w I am not prepared to say that the results deduced for the parameters a and b will be strictly the most probable ones according to the principles of the method of least squares.

LEHIGH UNIVERSITY, *October*, 1891.

A NEW ITALIAN MATHEMATICAL JOURNAL.

Rivista di Matematica, diretta da G. PEANO. Torino, Fratelli Bocca, 1891.

ALMOST simultaneously with the *Bulletin of the New York Mathematical Society*, a new journal of a somewhat similar character has been founded in Italy. Like the *Bulletin*, the *Rivista di Matematica* is a monthly of at least sixteen pages 8vo. According to the prospectus "its scope is essentially didactic, its principal object being the improvement of the methods of teaching." The *Rivista* will contain "articles and discussions concerning the fundamental principles of the science and also the history of mathematics." "The review of text-books and all publications having reference to the teaching of mathematics will form an important feature." Questions and inquiries about mathematical subjects sent to the editor will be either answered directly or published in the