

# THE ANNALS of STATISTICS

AN OFFICIAL JOURNAL OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS

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Vol. 1, No. 6—November 1973

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**The Annals of Statistics**, Volume 1, Number 6, November 1973. Published bimonthly in January, March, May, July, September, and November by The Institute of Mathematical Statistics, MR IV—Room 131, University of California, S.F., San Francisco, California 94122.

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Mail to the *Annals of Statistics* should be addressed to either the Editor, Managing Editor or the Treasurer as described above. It should *not* be addressed to Waverly Press.

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PRINTED AT THE

WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage paid at San Francisco, California and at additional mailing offices

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VOLUME 1

DECEMBER 1973

No. 6

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- Subadditive ergodic theory.....J. F. C. KINGMAN  
Discussion on Professor Kingman's paper by D. L. Burkholder,  
Daryl Daley, H. Kesten, P. Ney, Frank Spitzer and J. M. Hammersley

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- Another note on the Borel-Cantelli lemma and the strong law with the  
Poisson approximation as a by-product.....DAVID FREEDMAN  
Self-annihilating branching processes.....BRUCE K. ERICKSON  
The multitype Galton-Watson process with immigration  
NORMAN KAPLAN  
The law of the iterated logarithm for Gaussian processes  
HIROSHI OODAIRA  
Continuity of Gaussian processes and random Fourier series  
M. B. MARCUS  
Ergodic behavior for nonnegative kernels  
RICHARD W. MADSEN AND PATRICIA S. CONN  
Limit theorems for reversible Markov processes  
MICHAEL L. LEVITAN AND LAWRENCE H. SMOLOWITZ  
Local variation of diffusion in local time.....FRANK B. KNIGHT  
A method for computing the asymptotic limit of a class of expected first  
passage times.....WALTER A. ROSENKRANTZ  
Weak convergence of superpositions of randomly selected partial sums  
RICHARD F. SERFOZO

## Short Communications

- A Tauberian theorem of E. Landau and W. Feller.....E. SENETA  
A note on martingale square functions.....JIA-ARNG CHAO  
A weak convergence theorem for Gaussian sequences  
CHANDRAKANT M. DEO  
The growth of supercritical branching processes with random environments  
NIELS KEIDING AND JOHN E. NIELSEN  
A bound on tail probabilities for quadratic forms in independent random  
variables whose distributions are not necessarily symmetric  
F. T. WRIGHT  
An estimate for  $E(|S_n|)$  for variables in the domain of normal attraction  
of a stable law of index  $\alpha$ ,  $1 < \alpha < 2$ .....WILLIS L. OWEN