THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

Articles

Renewal theory for functionals of a Markov chain with general state	
space	355
Berry-Esséen estimates in Hilbert space and an application to the law	
of the iterated logarithm J. KUELBS AND T. KURTZ	387
of the iterated logarithm J. KUELBS AND T. KURTZ Sensitive discount optimality in controlled one dimensional diffusions	
Madrin I. Purdenan	408
Sojourn time problems Lajos Tákacs	420
Sojourn time problems LAJOS TÁKACS Limit theorems for direct sums Tze Leung LAI	432
Some results about multidimensional branching processes with random	
environments	441
environments	456
On quasi-compact Markov operators	464
•	
Short Communications	
A note on separable stochastic processes. Conditional distributions and tightness. Uniform inequalities for conditional expectations Convexity and conditional expectations D. RAMACHANDRAN	476
Conditional distributions and tightness. Patrick Billingsley	480
Uniform inequalities for conditional expectations L. Rogge	486
Convexity and conditional expectations J. PFANZAGL	490
Mixtures of perfect probability measures D. RAMACHANDRAN	495
Generalized distribution functions: the linearly ordered case with ap-	
plications to nonparametric statistics	501
A note on the supercritical branching processes with random environ-	
ments	-509
Asymptotic distributions for occupancy and waiting time problems	
with positive probability of falling through the cells . Lester Samuel-Cahn L_1 bounds for asymptotic normality of m -dependent sums using Stein's	515
L_1 bounds for asymptotic normality of m-dependent sums using Stein's	
technique	522
On the approximation of stationary measures by periodic and ergodic	
measures	530
measures	
pendent Gaussian processes	535
pendent Gaussian processes	540
On the divergence of a certain random series	
L. H. KOOPMANS, N. MARTIN, P. K. PATHAK AND C. QUALLS	546
On almost sure convergence of quadratic Browian variation	
W FERNANDEZ DE LA VEGA	551

Vol. 2, No. 3—June 1974

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics

OFFICERS

President:

R. R. Bahadur, Indian Statistical Institute, 538 Yojana Bhavan, Parliament Street, New Delhi-1, India.

President-Elect:

Frederick Mosteller, Department of Statistics, Science Center, Room 603, Harvard University, One Oxford Street, Cambridge, Massachusetts 02138.

Executive Secretary:

George J. Resnikoff, Dean of Graduate Studies, California State University, Hayward, 25800 Hillary Street, Hayward, California 94542.

Program Secretary:

R. V. Hogg, Department of Statistics, University of Iowa, Iowa City, Iowa 52240

Treasurer

Robert Elashoff, MR IV—Room 131, University of California, San Francisco, California 94122

Editor: Annals of Statistics

I. R. Savage, Department of Statistics, Florida State University, Tallahassee, Florida 32306

Editor: Annals of Probability

Ronald Pyke, Department of Mathematics, University of Washington, Seattle Washington 98195

Managing Editor:

K. J. C. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina 27514

Membership. Membership dues including a subscription to one Annals and The Institute of Mathematical Statistics Bulletin are \$18.00 per year for residents of the United States or Canada and \$12.00 per year for residents of other countries. Special rates of \$9.00 per year are available to students. Rates in each category are one-third higher for members who wish both Annals as well as the Bulletin. Inquiries regarding membership in the Institute should be sent to the Treasurer of the Institute.

should be sent to the Treasurer of the Institute.

Subscription Rates. Current volumes (six issues per calendar year) of the Annals of Probability and the Annals of Statistics are each \$25.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$5.00. Back numbers of both Annals and the Annals of Mathematical Statistics (Volumes 1 through 43) may be purchased from the Treasurer.

The Annals of Probability, Volume 2, Number 3, June 1974. Published bimouthly in February, April, June August October, and December by The Institute of Mathematical Statistics, MR IV Room 181, University of California, San Francisco, California 94122.

Mail to the Annals of Probability should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press

PRINTED AT THE
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage paid at San Francisco, California and at additional mailing offices

EDITORIAL STAFF

Editor

RONALD PYKE

ASSOCIATE EDITORS

J. R. Blum
DAVID R. BRILLINGER
D. J. DALEY
R. M. Dudley
RONALD K. GETOOR
DAVID L. HANSON

DONALD L. IGLEHART
J. F. C. KINGMAN
ULRICH KRENGEL
A. W. Marshall
Peter Ney
WILLIAM E. PRUITT

J. Sethuraman
L. A. Shepp
Charles Stone
HENRY TEICHER
JOHN B. WALSH
MICHAEL J. WICHURA

Managing Editor K. J. C. Smith

PAST EDITORS OF THE ANNALS OF MATHEMATICAL STATISTICS

H. C. Carver, 1930-1938
S. S. Wilks, 1938-1949
T. W. Anderson, 1950-1952
E. L. LEHMANN, 1953-1955
T. E. HARRIS, 1955-1958

William Kruskal, 1958–1961 J. L. Hodges Jr., 1961–1964 D. L. Burkholder, 1964–1967 Z. W. Birnbaum, 1967–1970 Ingram Olkin, 1970–1972

EDITORIAL POLICY

The main aim of the Annals of Probability and the Annals of Statistics is to publish original contributions related to the theory of statistics and probability. The emphasis is on quality, importance and interest; formal novelty and mathematical correctness alone are not sufficient. Particularly appropriate for the Annals are important theoretical papers and applied contributions which either stimulate theoretical interest in important new problems or make substantial progress in existing applied areas. Of special interest are authoritative expository or survey articles, whether on theoretical areas of vigorous recent development, or on specific applications. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

AEROSPACE CORPORATION El Segundo, California

THE AMERICAN COLLEGE TESTING PROGRAM IOWA City, Iowa

ARIZONA STATE UNIVERSITY Tempe, Arizona

ARTHUR D. LITTLE, INC. Cambridge, Massachusetts

BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
Murray Hill, N. J.

BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS Bowling Green, Ohio

CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS Hayward, California

CASE WESTERN RESERVE UNIVERSITY, DE-PARTMENT OF MATHEMATICS Cleveland, Ohio

CATHOLIC UNIVERSITY OF AMERICA, MATHE-MATICS DEPT. Washington, D.C.

CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS Tallahassee, Florida

FORD MOTOR COMPANY, SCIENTIFIC LAB-ORATORIES Dearborn, Michigan

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES Warren, Michigan

GEORGE WASHINGTON UNIVERSITY, DE-PARTMENT OF STATISTICS Washington, D.C.

INDIANA UNIVERSITY, MATHEMATICS DEPT. Bloomington, Indiana

INTERNATIONAL BUSINESS MACHINES COR-PORATION Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY Ames, Iowa

JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIOSTATISTICS Baltimore, Maryland

KANSAS STATE UNIVERSITY, DEPARTMENT OF STATISTICS Manhattan, Kansas

KNOLLS ATOMIC POWER LABORATORY Schenectady, New York

MIAMI UNIVERSITY, DEPARTMENT OF MATH-EMATICS Oxford, Ohio 45056 MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS East Lansing, Michigan

NATIONAL SECURITY AGENCY, Fort George G. Meade, Maryland

NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS Evanston, Illinois

OHIO STATE UNIVERSITY, DIVISION OF STATISTICS Columbus, Ohio

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS Corvallis, Oregon

PENNSYLVANIA STATE UNIVERSITY, DE-PARTMENT OF STATISTICS University Park, Pennsylvania

PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS Princeton, New Jersey

PURDUE UNIVERSITY LIBRARIES Lafayette, Indiana

THE RAND CORPORATION Santa Monica, California

THE ROCKEFELLER UNIVERSITY New York, New York

Sandia Corporation, Sandia Base Albuquerque, New Mexico

SIMON FRASER UNIVERSITY, DEPARTMENT OF MATHEMATICS Burnaby, British Columbia, Canada

SOUTHERN ILLINOIS UNIVERSITY, MATHE-MATICAL STUDIES Edwardsville, Illinois

SOUTHERN METHODIST UNIVERSITY, DE-PARTMENT OF STATISTICS Dallas, Texas

STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY Stanford, California

STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS Amherst, New York

THE TOBACCO INSTITUTE Washington, D.C.

TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS Lubbock, Texas 79409

UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER Brea, California

UNITED STATES ARMY RESEARCH AND DE-VELOPMENT CENTER Aberdeen Proving Ground, Maryland UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS Edmonton, Alberta, Canada

UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS
Tucson, Arizona

UNIVERSITY OF BRITISH COLUMBIA, DE-PARTMENT OF MATHEMATICS Vancouver, B.C., Canada

UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT Calgary 44, Alberta, Canada

UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY Berkeley, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS Irvine, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS Santa Barbara, California

UNIVERSITY OF GUELPH, MATHEMATICS AND STATISTICS DEPARTMENT Guelph, Ontario, Canada

UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, DEPARTMENT OF MATHEMATICS Chicago, Illinois

UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.
Urbana, Illinois

UNIVERSITY OF IOWA, DIVISION OF MATHE-MATICAL SCIENCES IOWA City, IOWA

UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS Winnipeg-19, Manitoba, Canada

University of Maryland, Department of Mathematics

College Park, Maryland

UNIVERSITY OF MASSACHUSETTS, DEPARTMENT OF MATHEMATICS & STATISTICS

Amherst, Massachusetts

UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS Ann Arbor, Michigan

UNIVERSITY OF MINNESOTA, SCHOOL OF STATISTICS
Minneapolis, Minnesota

UNIVERSITY OF MISSOURI, DEPARTMENT OF STATISTICS Columbia, Missouri

UNIVERSITY OF MISSOURI AT ROLLA, DE-PARTMENT OF MATHEMATICS Rolla, Missouri

UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS
Montreal, Quebec, Canada

UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS & STATISTICS Albuquerque, New Mexico

UNIVERSITY OF NORTH CAROLINA, DEPART-MENT OF STATISTICS Chapel Hill, North Carolina

UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS
Ottawa, Ontario, Canada

University of Rochester Rochester, New York

UNIVERSITY OF WASHINGTON, DEPARTMENT OF MATHEMATICS
Seattle, Washington

UNIVERSITY OF WISCONSIN, MADISON, DE-PARTMENT OF STATISTICS Madison, Wisconsin

UNIVERSITY OF WISCONSIN, MILWAUKEE,
DEPARTMENT OF MATHEMATICS
Milwaukee, Wisconsin

WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Detroit, Michigan

WEST CHESTER STATE COLLEGE West Chester, Pennsylvania

WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES Pittsburgh, Pennsylvania

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the Annals of Probability (For current address, see the latest issue of the Annals.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will not normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law", depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary", followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—. Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to Mathematical Reviews Index to Volume 39, June 1970. (See, also, The Notices of the American Mathematical Society, June 1970, pp. 616-618, for more details.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

List of Symbols. A completely typewritten list of symbols, identified typographically, not mathematically, should be attached to the manuscript on a separate page. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay", etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent;

thus, (a + b)/(c + d) is preferred to $\frac{a + b}{c + d}$, and

 $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}.$ Also, $a^{b(c)}$ and $a_{b(c)}$ are pre-

ferred to a^{b_c} and a_{b_c} , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. Ann. Math. Statist. 15 229-282.

In textual material, the format "... Doob (1944)..." is normally preferred to "... Doob [5]...". Multiple references can be distinguished as "... Doob (1944a)...". Abbreviations for journals should be taken from Mathematical Reviews Index to Volume 40, 1970, pp. 1683-1702.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability* (For current address, see the latest issue of the *Annals.*)