

THE ANNALS *of* PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

Special Invited Paper

Refinements of the multidimensional central limit theorem and applications R. N. BHATTACHARYA

Articles

Speeds of convergence for the multidimensional central limit theorem	T. J. SWEETING	28
Random stopping preserves regular variation of process distributions	PRISCILLA GREENWOOD AND ITREL MONROE	42
Level crossings of a stochastic process with absolutely continuous sample paths	MICHAEL B. MARCUS	52
The distribution of leading digits and uniform distribution mod 1	PERSI DIACONIS	72
The condenser problem	K. L. CHUNG AND R. K. GETTOOR	82
Insensitivity of steady-state distributions of generalized semi-Markov processes. Part 1	R. SCHAASBERGER	87
Limit theorems for branching processes in a random environment	DAVID TANNY	100
Weak convergence to Brownian meander and Brownian excursion	RICHARD T. DURRETT, DONALD L. IGLEHART AND DOUGLAS R. MILLER	117

Short Communications

Functionals of Brownian meander and Brownian excursion	RICHARD T. DURRETT AND DONALD L. IGLEHART	130
On the Berry-Esseen theorem for U -statistics	Y.-K. CHAN AND JOHN WIERMAN	136
Wiener functionals as Itô integrals	R. M. DUDLEY	140
On the oscillation of the Brownian motion random measure	SIAMAK KHALILI	142
Hausdorff dimension and Gaussian fields	ROBERT J. ADLER	145
Last exit times from the boundary of a continuous time Markov chain	MICHAEL W. CHAMBERLAIN	152
An unexpected expectation	GORDON SIMONS	157

Note

Correction to "An integral equation in age-dependent branching processes"	H. WEINER	159
---	-----------	-----

Vol. 5, No. 1—February 1977

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:

C. R. Rao, Indian Statistical Institute, 7, S.J.S. Sansanwal Marg, New Delhi 110001, India

President-Elect:

Elizabeth Scott, Department of Statistics, University of California, Berkeley, California 94720

Past President:

Donald L. Burkholder, Department of Mathematics, University of Illinois, Urbana, Illinois 61801

Executive Secretary:

George J. Resnikoff, Department of Statistics, California State University, Hayward, 25800 Hillary Street, Hayward, California 94542

Treasurer:

Robert M. Elashoff, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward California 94545

Program Secretary:

William L. Harkness, Pennsylvania State University, University Park, Pennsylvania 16801

Editor: *Annals of Statistics*

Rupert G. Miller, Jr., Department of Statistics, Stanford University, Stanford, California 94305

Editor: *Annals of Probability*

Patrick Billingsley, Department of Statistics, University of Chicago, Chicago, Illinois 60637

Managing Editor:

Donald R. Truax, Department of Mathematics, University of Oregon, Eugene, Oregon 97403

Membership. Membership dues including a subscription to one *Annals* and *The Institute of Mathematical Statistics Bulletin* are \$24.00 per year for residents of the United States or Canada and \$16.00 per year for residents of other countries. Special rates of \$12.00 per year are available to students. Rates in each category are one-third higher for members who wish both *Annals* as well as the *Bulletin*. Inquiries regarding membership in the Institute should be sent to the Treasurer at the business office.

Subscription Rates. Current volumes (six issues per calendar year) of the *Annals of Probability* and the *Annals of Statistics* are each \$30.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$7.00. Back numbers of both *Annals* and the *Annals of Mathematical Statistics* (Volumes 1 through 43) may be purchased from the Treasurer.

The Annals of Probability. Volume 5, Number 1, February 1977. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

Mail to the *Annals of Probability* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

PRINTED AT THE
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage paid at Hayward, California and at additional mailing offices

Copyright © 1977 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

PATRICK BILLINGSLEY

ASSOCIATE EDITORS

RICHARD E. BARLOW
RABI BHATTACHARYA
J. R. BLUM
DAVID R. BRILLINGER
JEAN PIERRE CONZE
D. A. DAWSON
R. M. DUDLEY
MEYER DWASS

DAVID L. HANSON
C. C. HEYDE
DONALD L. IGLEHART
NARESH C. JAIN
SØREN JOHANSEN
J. F. C. KINGMAN
M. R. LEADBETTER
PETER NEY

WALTER PHILIPP
WILLIAM E. PRUITT
G. E. H. REUTER
L. A. SHEPP
DAVID O. SIEGMUND
CHARLES J. STONE
JOHN B. WALSH
MICHAEL J. WICHURA
DONALD YLVISSAKER

MANAGING EDITOR

D. R. TRUAX

PAST EDITORS
ANNALS OF MATHEMATICAL STATISTICS

H. C. CARVER, 1930-1938
S. S. WILKS, 1938-1949
T. W. ANDERSON, 1950-1952
E. L. LEHMANN, 1953-1955
T. E. HARRIS, 1955-1958

WILLIAM KRUSKAL, 1958-1961
J. L. HODGES, JR., 1961-1964
D. L. BURKHOLDER, 1964-1967
Z. W. BIRNBAUM, 1967-1970
INGRAM OLKIN, 1970-1972

ANNALS OF PROBABILITY
RONALD PYKE, 1972-1975

ANNALS OF STATISTICS
INGRAM OLKIN, 1972-1973
I. R. SAVAGE, 1974-1976

EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

- AEROSPACE CORPORATION
El Segundo, California
- ARIZONA STATE UNIVERSITY
Tempe, Arizona
- ARTHUR D. LITTLE, INC.
Cambridge, Massachusetts
- BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
Murray Hill, N. J.
- BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS
Bowling Green, Ohio
- CALIFORNIA STATE UNIVERSITY, FULLERTON, DEPARTMENT OF MATHEMATICS
Fullerton, California
- CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS
Hayward, California
- CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Cleveland, Ohio
- CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York
- FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Tallahassee, Florida
- FORD MOTOR COMPANY, ENGINEERING AND RESEARCH LIBRARY
Dearborn, Michigan
- GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES
Warren, Michigan
- GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS
Washington, D.C.
- INDIANA UNIVERSITY, MATHEMATICS DEPT.
Bloomington, Indiana
- INSTITUTE FOR DEFENSE ANALYSES
Arlington, Virginia
- INTERNATIONAL BUSINESS MACHINES CORPORATION
Armonk, New York
- IOWA STATE UNIVERSITY, STATISTICAL LABORATORY
Ames, Iowa
- JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatISTICS, DEPARTMENT OF MATHEMATICAL SCIENCES
Baltimore, Maryland
- KANSAS STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Manhattan, Kansas
- MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS
Oxford, Ohio
- MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY
East Lansing, Michigan
- NATIONAL INSTITUTE FOR THE DEAF, OFFICE FOR PROGRAM ANALYSIS
Rochester, New York
- NATIONAL SECURITY AGENCY
Fort George G. Meade, Maryland
- NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico
- NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS
Evanston, Illinois
- OHIO STATE UNIVERSITY, DIVISION OF STATISTICS
Columbus, Ohio
- OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Corvallis, Oregon
- PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
University Park, Pennsylvania
- PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS
Princeton, New Jersey
- PURDUE UNIVERSITY LIBRARIES
Lafayette, Indiana
- QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS, KINGSTON
Ontario, Canada
- RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Houston, Texas
- THE ROCKEFELLER UNIVERSITY
New York, New York
- SANDIA CORPORATION, SANDIA BASE
Albuquerque, New Mexico
- SOUTHERN ILLINOIS UNIVERSITY, MATHEMATICAL STUDIES
Edwardsville, Illinois
- SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS
Dallas, Texas
- STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY
Stanford, California
- STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS
Amherst, New York
- TEMPLE UNIVERSITY, MATHEMATICS DEPARTMENT
Philadelphia, Pennsylvania
- TEXAS A & M UNIVERSITY, DEPT. OF MATHEMATICS
College Station, Texas

TEXAS TECH UNIVERSITY, DEPARTMENT OF
MATHEMATICS
Lubbock, Texas 79409

THE TOBACCO INSTITUTE
Washington, D.C.

UNION OIL COMPANY OF CALIFORNIA, UNION
RESEARCH CENTER
Brea, California

UNITED STATES ARMY RESEARCH AND DE-
VELOPMENT CENTER
Aberdeen Proving Ground, Maryland

UNIVERSITY OF ALBERTA, DEPARTMENT OF
MATHEMATICS
Edmonton, Alberta, Canada

UNIVERSITY OF ARIZONA, DEPARTMENT OF
MATHEMATICS
Tucson, Arizona

UNIVERSITY OF BRITISH COLUMBIA, DE-
PARTMENT OF MATHEMATICS
Vancouver, B.C., Canada

UNIVERSITY OF CALGARY, MATHEMATICS
DEPARTMENT
Calgary 44, Alberta, Canada

UNIVERSITY OF CALIFORNIA, BERKELEY,
STATISTICAL LABORATORY
Berkeley, California

UNIVERSITY OF CINCINNATI, DEPARTMENT
OF MATHEMATICAL SCIENCES
Cincinnati, Ohio

UNIVERSITY OF GUELPH, MATHEMATICS AND
STATISTICS DEPARTMENT
Guelph, Ontario, Canada

UNIVERSITY OF ILLINOIS AT CHICAGO
CIRCLE, DEPARTMENT OF MATHEMATICS
Chicago, Illinois

UNIVERSITY OF ILLINOIS, MATHEMATICS
DEPT.
Urbana, Illinois

UNIVERSITY OF IOWA, DIVISION OF MATHE-
MATICAL SCIENCES
Iowa City, Iowa

UNIVERSITY OF MANITOBA, DEPARTMENT
OF STATISTICS
Winnipeg, Manitoba, Canada

UNIVERSITY OF MARYLAND, DEPARTMENT
OF MATHEMATICS
College Park, Maryland

UNIVERSITY OF MICHIGAN, DEPARTMENT OF
STATISTICS
Ann Arbor, Michigan

UNIVERSITY OF MINNESOTA, SCHOOL OF
STATISTICS
Minneapolis, Minnesota

UNIVERSITY OF MISSOURI, DEPARTMENT OF
STATISTICS
Columbia, Missouri

UNIVERSITY OF MISSOURI AT ROLLA, DE-
PARTMENT OF MATHEMATICS
Rolla, Missouri

UNIVERSITY OF MONTREAL, DEPARTMENT
OF MATHEMATICS
Montreal, Quebec, Canada

UNIVERSITY OF NEW MEXICO, DEPARTMENT
OF MATHEMATICS AND STATISTICS
Albuquerque, New Mexico

UNIVERSITY OF NORTH CAROLINA, DEPART-
MENT OF STATISTICS
Chapel Hill, North Carolina

UNIVERSITY OF OREGON, MATHEMATICS DE-
PARTMENT
Eugene, Oregon

UNIVERSITY OF OTTAWA, DEPARTMENT OF
MATHEMATICS
Ottawa, Ontario, Canada

UNIVERSITY OF SOUTH CAROLINA, DEPART-
MENT OF MATHEMATICS AND COMPUTER
SCIENCE
Columbia, South Carolina

UNIVERSITY OF TEXAS, DEPARTMENT OF
MATHEMATICS
Austin, Texas

UNIVERSITY OF UTAH, DEPT. OF MATHEMAT-
ICS
Salt Lake City, Utah

UNIVERSITY OF VICTORIA, DEPT. OF MATH-
EMATICS
Victoria, British Columbia, Canada

UNIVERSITY OF WASHINGTON, DEPARTMENT
OF MATHEMATICS
Seattle, Washington

UNIVERSITY OF WISCONSIN, MADISON, DE-
PARTMENT OF STATISTICS
Madison, Wisconsin

UNIVERSITY OF WISCONSIN, MILWAUKEE,
DEPARTMENT OF MATHEMATICS
Milwaukee, Wisconsin

VIRGINIA COMMONWEALTH UNIVERSITY, DE-
PARTMENT OF MATHEMATICAL SCIENCES
Richmond, Virginia

WAYNE STATE UNIVERSITY, DEPARTMENT
OF MATHEMATICS
Detroit, Michigan

WEST CHESTER STATE COLLEGE
West Chester, Pennsylvania

WESTINGHOUSE ELECTRIC CORPORATION,
RESEARCH LABORATORIES
Pittsburgh, Pennsylvania

YALE UNIVERSITY, STATISTICS DEPART-
MENT
New Haven, Connecticut

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will *not* normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—. Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to *Mathematical Reviews* Index to Volume 39, June 1970. (See, also, *The Notices of the American Mathematical Society*, June 1970; *Bulletin of the Institute of Mathematical Statistics*, September 1974; or a current index issue of *Mathematical Reviews*.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

Identification of Symbols. Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent; thus, $(a + b)/(c + d)$ is preferred to $\frac{a + b}{c + d}$, and $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}$. Also, $a^{b(c)}$ and $a_{b(c)}$ are pre-

ferred to a^{bc} and a_{bc} , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. *Ann. Math. Statist.* 15 229-282.

In textual material, the format "... Doob (1944) ..." is normally preferred to "... Doob [5] ..." Multiple references can be distinguished as "... Doob (1944a) ..." Abbreviations for journals should be taken from a current index issue of *Mathematical Reviews*.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability* (For current address, see the latest issue of the *Annals*.)