

THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF
THE INSTITUTE OF MATHEMATICAL STATISTICS

Articles

Clustering and dispersion rates for some interacting particle systems on \mathbb{Z}	MAURY BRAMSON AND DAVID GRIFFEATH	183
A strong law for some generalized urn processes	BRUCE HILL, DAVID LANE AND WILLIAM SUDDERTH	214
Bounds, inequalities, and monotonicity properties for some specialized renewal processes	MARK BROWN	227
Limit distributions for the error in approximations of stochastic integrals	HOLGER ROOTZÉN	241
A limit theorem for the norm of random matrices	STUART GEMAN	252
Matrix normalization of sums of random vectors in the domain of attraction of the multivariate normal	MARJORIE G. HAHN AND MICHAEL J. KLASS	262
Speeds of convergence and asymptotic expansions in the central limit theorem: a treatment by operators	T. J. SWEETING	281
Convergence rates for probabilities of moderate deviations for sums of random variables with multidimensional indices	ALLAN GUT	298
Limit theorems without moment hypotheses for sums of independent random variables	R. J. TOMKINS	314
Rates of escape of infinite dimensional Brownian motion	K. BRUCE ERICKSON	325
A characterization of Vitali conditions in terms of maximal inequalities	ANNIE MILLET AND LOUIS SUCHESTON	339
Precision bounds for the relative error in approximation of $E S_n $ and extensions	MICHAEL J. KLASS	350

Short Communications

Monotonicity of an integral of M. Klass	JAMES REEDS	368
On Kamae's conjecture concerning the d -distance between two-state Markov processes	MARTIN H. ELLIS	372
A new look at convergence of branching processes	D. R. GREY	377
Exponential moments of vector valued random series and triangular arrays	ALEJANDRO DE ACOSTA	381
Limit points associated with generalized iterated logarithm laws	LUISA TURRIN FERNHÖLZ	390
Almost sure limit points of maxima of stationary Gaussian sequences	H. VISHNU HEBBAR	393
Zeros of the densities of infinitely divisible measures	PATRICK L. BROCKETT AND WILLIAM N. HUDSON	400

Vol. 8, No. 2—April 1980

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:

George E. P. Box, Department of Statistics, University of Wisconsin, 1210 W. Dayton St., Madison, Wisconsin 53706

President-Elect:

Peter Bickel, Department of Statistics, University of California, Berkeley, California 94720

Past-President:

Samuel Karlin, Department of Mathematics, Stanford University, Stanford, California 94305

Executive Secretary:

Martin Fox, Department of Statistics and Probability, Michigan State University, East Lansing, Michigan 48824

Treasurer:

Heebok Park, Department of Statistics, California State University, Hayward, California 94542
IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545

Program Secretary:

William L. Harkness, Pennsylvania State University, University Park, Pennsylvania 16801

Editor: *Annals of Statistics*

David V. Hinkley, Department of Applied Statistics, University of Minnesota, St. Paul Campus, St. Paul, Minnesota 55108

Editor: *Annals of Probability*

R. M. Dudley, Department of Mathematics, Room 2-245, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

Managing Editor:

Donald R. Truax, Department of Mathematics, University of Oregon, Eugene, Oregon 97403.

Membership. Membership dues including a subscription to one *Annals* and *The Institute of Mathematical Statistics Bulletin* are \$32 per year for all members. Special rates of \$15.00 per year are available to students. Rates in each category are one-third higher for members who wish both *Annals* as well as the *Bulletin*. Inquiries regarding membership in the institute should be sent to the Treasurer at the business office.

Subscription Rates. Current volumes (six issues per calendar year) of the *Annals of Probability* are \$40.00. Current volumes (six issues per calendar year) of the *Annals of Statistics* are \$45.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$8.00. Back numbers of both *Annals* and the *Annals of Mathematical Statistics* (Volumes 1 through 43) may be purchased from the Treasurer.

The Annals of Probability, Volume 8, Number 2, April 1980. Published bimonthly in February, April, June, August, October, and December by *The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.*

Mail to the *Annals of Probability* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

Printed at the
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage at Hayward, California and at additional mailing offices

Copyright © 1980 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

R. M. DUDLEY

ASSOCIATE EDITORS

ALEXANDRA BELLOW
SIMEON M. BERMAN
RABI BHATTACHARYA
MIKLÓS CSÖRGŐ
C. A. DOLÉANS-DADE
VÁCLAV FABIAN

C. C. HEYDE
NARESH C. JAIN
MARK KAC
HARRY KESTEN
THOMAS G. KURTZ
M. R. LEADBETTER

THOMAS M. LIGGETT
WALTER PHILIPP
STANLEY SAWYER
DAVID O. SIEGMUND
LAJOS TAKÁCS
JOHN B. WALSH

EDITORIAL ASSISTANT

CYNTHIA FRIEDMAN

MANAGING EDITOR

D. R. TRUAX

EDITORIAL ASSISTANTS

RALPH KRUMDIECK

ESTHER ELLEN NOVITSKI

PAST EDITORS ANNALS OF MATHEMATICAL STATISTICS

H. C. CARVER, 1930–1938
S. S. WILKS, 1938–1949
T. W. ANDERSON, 1950–1952
E. L. LEHMANN, 1953–1955
T. E. HARRIS, 1955–1958

WILLIAM KRUSKAL, 1958–1961
J. L. HODGES, JR., 1961–1964
D. L. BURKHOLDER, 1964–1967
Z. W. BIRNBAUM, 1967–1970
INGRAM OLKIN, 1970–1972

ANNALS OF PROBABILITY
RONALD PYKE, 1972–1975
PATRICK BILLINGSLEY, 1976–1978

ANNALS OF STATISTICS
INGRAM OLKIN, 1972–1973
I. R. SAVAGE, 1974–1976
RUPERT G. MILLER, JR., 1977–1979

EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

- AEROSPACE CORPORATION**
El Segundo, California
- ARIZONA STATE UNIVERSITY**
Tempe, Arizona
- BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY**
Murray Hill, N.J.
- BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS**
Bowling Green, Ohio
- CALIFORNIA STATE UNIVERSITY, FULLERTON, DEPARTMENT OF MATHEMATICS**
Fullerton, California
- CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS**
Hayward, California
- CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Cleveland, Ohio
- CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Ithaca, New York
- FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
Tallahassee, Florida
- FORD MOTOR COMPANY, ENGINEERING AND RESEARCH LIBRARY**
Dearborn, Michigan
- GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES**
Warren, Michigan
- GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS**
Washington, D.C.
- INDIANA UNIVERSITY, MATHEMATICS DEPT.**
Bloomington, Indiana
- INSTITUTE FOR DEFENSE ANALYSES**
Arlington, Virginia
- INTERNATIONAL BUSINESS MACHINES CORPORATION**
Armonk, New York
- IOWA STATE UNIVERSITY, STATISTICAL LABORATORY**
Ames, Iowa
- JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatISTICS, DEPARTMENT OF MATHEMATICAL SCIENCES**
Baltimore, Maryland
- KANSAS STATE UNIVERSITY, STATISTICS DEPARTMENT**
Manhattan, Kansas
- MARQUETTE UNIVERSITY, MATHEMATICS AND STATISTICS DEPARTMENT**
Milwaukee, Wisconsin
- MASSACHUSETTS INSTITUTE OF TECHNOLOGY MATHEMATICS DEPARTMENT**
Cambridge, Massachusetts
- MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Oxford, Ohio
- MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY**
East Lansing, Michigan
- NATIONAL SECURITY AGENCY**
Fort George G. Meade, Maryland
- NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
Las Cruces, New Mexico
- NORTHERN ILLINOIS UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
De Kalb, Illinois
- NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Evanston, Illinois
- OHIO STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
Columbus, Ohio
- OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
Corvallis, Oregon
- PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF STATISTICS**
University Park, Pennsylvania
- PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS**
Princeton, New Jersey
- PURDUE UNIVERSITY LIBRARIES**
Lafayette, Indiana
- QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS, KINGSTON**
Ontario, Canada
- RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
Houston, Texas
- THE ROCKEFELLER UNIVERSITY**
New York, New York
- SANDIA CORPORATION, SANDIA BASE**
Albuquerque, New Mexico
- SIMON FRASER UNIVERSITY, MATHEMATICS DEPARTMENT**
Burnaby, Canada
- SOUTHERN ILLINOIS UNIVERSITY, MATHEMATICAL STUDIES**
Edwardsville, Illinois
- SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS**
Dallas, Texas
- STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY**
Stanford, California
- STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS**
Amherst, New York
- TEMPLE UNIVERSITY, MATHEMATICS DEPARTMENT**
Philadelphia, Pa
- TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Lubbock, Texas 79409

- THE TOBACCO INSTITUTE**
Washington, D.C.
- UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER**
Brea, California
- UNITED STATES ARMY RESEARCH AND DEVELOPMENT CENTER**
Aberdeen Proving Ground, Maryland
- UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS**
Edmonton, Alberta, Canada
- UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS AND COMMITTEE ON STATISTICS**
Tucson, Arizona
- UNIVERSITY OF BRITISH COLUMBIA, DEPARTMENT OF MATHEMATICS**
Vancouver, B.C., Canada
- UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT**
Calgary 44, Alberta, Canada
- UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY**
Berkeley, California
- UNIVERSITY OF CINCINNATI, DEPARTMENT OF MATHEMATICAL SCIENCES**
Cincinnati, Ohio
- UNIVERSITY OF GUELPH, MATHEMATICS AND STATISTICS DEPARTMENT**
Guelph, Ontario, Canada
- UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, DEPARTMENT OF MATHEMATICS**
Chicago, Illinois
- UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.**
Urbana, Illinois
- UNIVERSITY OF IOWA, DIVISION OF MATHEMATICAL SCIENCES**
Iowa City, Iowa
- UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS**
Winnipeg, Manitoba, Canada
- UNIVERSITY OF MARYLAND, DEPARTMENT OF MATHEMATICS**
College Park, Maryland
- UNIVERSITY OF MASSACHUSETTS, MATHEMATICS AND STATISTICS DEPARTMENT**
Amherst, Massachusetts
- UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS**
Ann Arbor, Michigan
- UNIVERSITY OF MINNESOTA, SCHOOL OF STATISTICS**
Minneapolis, Minnesota
- UNIVERSITY OF MISSOURI, DEPARTMENT OF STATISTICS**
Columbia, Missouri
- UNIVERSITY OF MISSOURI AT ROLLA, DEPARTMENT OF MATHEMATICS**
Rolla, Missouri
- UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS**
Montreal, Quebec, Canada
- UNIVERSITY OF NEBRASKA, MATHEMATICS AND STATISTICS DEPARTMENT**
Lincoln, Nebraska
- UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS AND STATISTICS**
Albuquerque, New Mexico
- UNIVERSITY OF NORTH CAROLINA, DEPARTMENT OF STATISTICS**
Chapel Hill, North Carolina
- UNIVERSITY OF OREGON, MATHEMATICS DEPARTMENT**
Eugene, Oregon
- UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS**
Ottawa, Ontario, Canada
- UNIVERSITY OF ROCHESTER, LIBRARY**
Rochester, New York
- UNIVERSITY OF SOUTH CAROLINA, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE**
Columbia, South Carolina
- UNIVERSITY OF TEXAS, DEPARTMENT OF MATHEMATICS**
Austin, Texas
- UNIVERSITY OF TEXAS, MATHEMATICS DEPT.**
San Antonio, Texas
- UNIVERSITY OF UTAH, DEPT. OF MATHEMATICS**
Salt Lake City, Utah
- UNIVERSITY OF VICTORIA, DEPT. OF MATHEMATICS**
Victoria, British Columbia, Canada
- UNIVERSITY OF WASHINGTON, DEPARTMENT OF MATHEMATICS**
Seattle, Washington
- UNIVERSITY OF WATERLOO, STATISTICS DEPARTMENT**
Waterloo, Ont., Canada
- UNIVERSITY OF WISCONSIN, MADISON, DEPARTMENT OF STATISTICS**
Madison, Wisconsin
- UNIVERSITY OF WISCONSIN, MILWAUKEE, DEPARTMENT OF MATHEMATICS**
Milwaukee, Wisconsin
- VIRGINIA COMMONWEALTH UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES**
Richmond, Virginia
- WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS**
Detroit, Michigan
- WEST CHESTER STATE COLLEGE**
West Chester, Pennsylvania
- WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES**
Pittsburgh, Pennsylvania

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will *not* normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—.
Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to *Mathematical Reviews* Index to Volume 39, June 1970. (See, also, *The Notices of the American Mathematical Society*, June 1970; *Bulletin of the Institute of Mathematical Statistics*, September 1974; or a current index issue of *Mathematical Reviews*.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

Identification of Symbols. Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent;

thus, $(a + b)/(c + d)$ is preferred to $\frac{a + b}{c + d}$, and $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}$. Also, $a^{b(c)}$ and $a_{b(c)}$ are pre-

ferred to a^{bc} and a_{bc} , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. *Ann. Math. Statist.* 15 229–282.

In textual material, the format "... Doob (1944) ..." is normally preferred to "...Doob [5]..." Multiple references can be distinguished as "... Doob (1944a)..." Abbreviations for journals should be taken from a current index issue of *Mathematical Reviews*.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the *Annals of Probability*. (For current address, see the latest issue of the *Annals*.)