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CORRECTION

RANDOM WALK IN A RANDOM ENVIRONMENT AND FIRST-PASSAGE PERCOLATION ON TREES

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The proofs of Proposition 2 and of the first two parts of Theorem 3(ii) are incorrect, although the results themselves are correct. Here are correct proofs.

PROOF OF PROPOSITION 2. Let Π_n be as indicated. Our assumption is that

$$M := \sup_n \mathbf{E}\left[\sum_{\sigma\in\Pi_n} C_\sigma^x\right] = \sup_n \sum_{\sigma\in\Pi_n} p^{|\sigma|} < \infty.$$

Since $x \geq -1$, the inequality between the harmonic mean and the power mean of order x states that for positive numbers a_n, we have

$$\left(\frac{1}{N}\sum_{n=1}^N a_n^{-1}\right)^{-1} \leq \left(\frac{1}{N}\sum_{n=1}^N a_n^x\right)^{1/x}.$$

Take the xth power of both sides, use $a_n := \sum_{\sigma\in\Pi_n} C_\sigma$ and the fact that $a_n^x \leq \sum_{\sigma\in\Pi_n} C_\sigma^x$ since $0 < x \leq 1$ to obtain

$$\left(\frac{1}{N}\sum_{n=1}^N\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1}\right)^{-x} \leq \frac{1}{N}\sum_{n=1}^N\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^x \leq \frac{1}{N}\sum_{n=1}^N\sum_{\sigma\in\Pi_n} C_\sigma^x.$$

Now take the expectation to arrive at the bound

$$\mathbf{E}\left[\left(\frac{1}{N}\sum_{n=1}^N\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1}\right)^{-x}\right] \leq \mathbf{E}\left[\frac{1}{N}\sum_{n=1}^N\sum_{\sigma\in\Pi_n} C_\sigma^x\right] \leq M.$$

Fix $L > 0$. By Markov's inequality, we may deduce that

$$\begin{aligned}\mathbf{P}\left[\sum_{n=1}^\infty\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1} \leq L\right] &\leq \mathbf{P}\left[\sum_{n=1}^N\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1} \leq L\right] \\ &= \mathbf{P}\left[\left(\frac{1}{N}\sum_{n=1}^N\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1}\right)^{-x} \geq \left(\frac{N}{L}\right)^x\right] \\ &\leq M\left(\frac{L}{N}\right)^x.\end{aligned}$$

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Since this holds for all N, we conclude that

$$\mathbf{P}\left[\sum_{n=1}^{\infty}\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1} \le L\right] = 0,$$

and since this holds for all L, it follows that

$$\sum_{n=1}^{\infty}\left(\sum_{\sigma\in\Pi_n} C_\sigma\right)^{-1} = \infty$$

a.s., which implies recurrence by the Nash–Williams criterion ([4], Corollary 4.2.) □

PROOF OF FIRST TWO PARTS OF THEOREM 3(ii). This proof is quite similar, but starts from the hypothesis

$$\mathbf{E}\left[\sum_{|\sigma|=n} C_\sigma^x\right] = m^n p^n \le 1.$$

The above argument then gives

$$\sum_{n=1}^{\infty}\left(\sum_{|\sigma|=n} C_\sigma\right)^{-1} = \infty$$

a.s., which allows us to complete the proof as before. □

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