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ASSOCIATED LIE ALGEBRAS AND GRADED CONTRACTIONS OF THE PAULI GRADED $\mathfrak{sl}(3, \mathbb{C})$

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Abstract. We consider the Pauli grading of the Lie algebra $\mathfrak{sl}(3, \mathbb{C})$ and use the concept of graded contractions to construct non-isomorphic Lie algebras of dimension eight. We overview methods used to distinguish the results and show how associated algebras, uniquely determined by the original algebra, simplify this task. We present a short overview of resulting Lie algebras.

1. Introduction

Simple Lie algebra $A_2 = \mathfrak{sl}(3, \mathbb{C})$ as well as its real forms and its corresponding gradings, have found numerous applications in physics. Recall that a decomposition of a finite-dimensional Lie algebra $\mathcal{L}$ into a direct sum of its subspaces $\mathcal{L}_i, i \in I$

$$\mathcal{L} = \bigoplus_{i \in I} \mathcal{L}_i \tag{1}$$

is called a **grading**, when for all i, j from some set I there exists $k \in I$ such that

$$[\mathcal{L}_i, \mathcal{L}_j] \subseteq \mathcal{L}_k. \tag{2}$$

The grading $\Gamma : \mathcal{L} = \bigoplus_{i \in I} \mathcal{L}_i$ is a **refinement** of the grading $\tilde{\Gamma} : \mathcal{L} = \bigoplus_{j \in J} \tilde{\mathcal{L}}_j$ if for each $i \in I$ there exists $j \in J$ such that $\mathcal{L}_i \subseteq \tilde{\mathcal{L}}_j$. Refinement is called **proper** if the cardinality of I is greater than the cardinality of the set J. Grading which cannot be properly refined is called **fine**. The property (2) defines a binary operation on the set I. If $[\mathcal{L}_i, \mathcal{L}_j] = \{0\}$ holds, we can choose an arbitrary k. It is proved in [8] that for simple Lie algebras the index set I with this operation can always be embedded into an **Abelian group** G; then we say that the Lie algebra is graded by the group G, which is called a **grading group**. Fine gradings of simple Lie algebras are analogous of Cartan's root decomposition. On the physical side, they yield quantum observables with additive quantum numbers.

Relations between Lie algebras of the same dimension can be studied by means of contractions or deformations. Special place belongs to graded contractions – contractions which preserve a given grading and in this way generate further Lie algebras with the same additive quantum numbers.

Since the classification of complex and real Lie algebras of low dimensions 3, 4 and 5 is known, the obtained graded contractions in these dimensions are classified. This is the case e.g. for graded contractions of $A_1 = \mathfrak{sl}(2,\mathbb{C})$ and its real forms in [10]. The outcome of the Pauli graded $\mathfrak{sl}(2,\mathbb{C})$ are Euclidean and Heiseinberg algebras.

Besides the root decomposition, A_2 has 3 fine gradings [4], and the full graph of its gradings consists of 17 gradings including the 4 fine gradings and the trivial grading – the whole A_2. In order to obtain all graded contractions of A_2, it is sufficient to consider only the fine gradings, since among them all contractions are found which would come from the coarse gradings.

Graded contractions for the root decomposition were the most recently obtained in [1]. The investigations resulted there in 34 Lie algebras, of which nine are eight-dimensional non-decomposable. The present study is devoted to graded contractions of A_2 for so-called Pauli grading.

2. Graded Contractions of the Pauli Graded $\mathfrak{sl}(3,\mathbb{C})$

The Pauli grading of $\mathfrak{sl}(3,\mathbb{C})$ may be considered as a generalization of 2×2 Pauli matrices [7]. We list this $\mathbb{Z}_3 \times \mathbb{Z}_3$ grading explicitly. If we define matrices P, Q by the formula

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \qquad Q = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

then the Pauli grading has the following form

$$\begin{aligned} \mathfrak{sl}(3,\mathbb{C}) &= l_{01} \oplus l_{02} \oplus l_{10} \oplus l_{20} \oplus l_{11} \oplus l_{22} \oplus l_{12} \oplus l_{21} \\ &= Q \oplus Q^2 \oplus P \oplus P^2 \oplus PQ \oplus P^2Q^2 \oplus PQ^2 \oplus P^2Q \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \oplus \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \\ &\quad \oplus \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega^2 \\ 1 & 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & \omega \\ 1 & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \end{aligned} \tag{3}$$

where $\omega = \exp(2\pi\mathrm{i}/3)$. Note that the symbol for linear span was omitted.

Introducing **contraction parameters** $\varepsilon_{ij} \in \mathbb{C}$ we define a bilinear mapping $[\ ,\ ]_\varepsilon$ on the underlying vector space of $\mathfrak{sl}(3,\mathbb{C})$ by the formula

$$[x,y]_\varepsilon := \varepsilon_{ij}[x,y] \quad \text{for all} \quad x \in \mathcal{L}_i,\ y \in \mathcal{L}_j, \qquad i,j \in \mathbb{Z}_3 \times \mathbb{Z}_3. \tag{4}$$

If $[\ ,\ ]_\varepsilon$ is a Lie product, then it is called a **graded contraction** of the Lie algebra $\mathfrak{sl}(3,\mathbb{C})$. Such defined graded contraction preserves the Pauli grading because it is also true that $[\ ,\ ]_\varepsilon$ is $\mathbb{Z}_3 \times \mathbb{Z}_3$ graded. There are two types of conditions which parameters ε_{ij} must fulfill and the antisymmetry immediately gives

$$\varepsilon_{ij} = \varepsilon_{ji} \tag{5}$$

while Jacobi identities imply 48 equations

$$\begin{aligned} &\varepsilon_{(02)(10)A}\varepsilon_{(01)(12)A} - \varepsilon_{(01)(10)A}\varepsilon_{(02)(11)A} = 0 \\ &\varepsilon_{(10)(11)A}\varepsilon_{(01)(21)A} - \varepsilon_{(01)(11)A}\varepsilon_{(10)(12)A} = 0 \quad \text{for all } A \in \mathrm{SL}(2,\mathbb{Z}_3) \end{aligned} \tag{6}$$

where we used abbreviation $\varepsilon_{(ij)(kl)A} := \varepsilon_{(ij)A,(kl)A}$ and

$$\mathrm{SL}(2,\mathbb{Z}_3) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ;\ a,b,c,d \in \mathbb{Z}_3,\ ad - bc = 1 \bmod 3 \right\}.$$

Any solution of the equations (5), (6) determines certain eight-dimensional Lie algebra. However, it might be difficult to decide whether two of such Lie algebras are isomorphic or not.

3. Associated Lie Algebras

Let us take for example two solutions of (5), (6) which lead to two Lie algebras $\mathcal{L}, \widetilde{\mathcal{L}}$. The non-zero commutation relations of these graded contractions of the Pauli graded $\mathfrak{sl}(3,\mathbb{C})$ are

$$\begin{array}{llllll} \mathcal{L} & [l_{01},l_{10}] = l_{11}, & [l_{01},l_{20}] = l_{21}, & [l_{01},l_{11}] = l_{12}, & [l_{01},l_{22}] = l_{20} \\ & [l_{02},l_{10}] = l_{12}, & [l_{10},l_{11}] = l_{21}, & [l_{20},l_{22}] = l_{12} & \end{array}$$

$$\begin{array}{llllll} \widetilde{\mathcal{L}} & [l_{01},l_{10}] = l_{11}, & [l_{01},l_{20}] = l_{21}, & [l_{01},l_{22}] = l_{20}, & [l_{02},l_{10}] = l_{12} \\ & [l_{02},l_{22}] = l_{21}, & [l_{10},l_{11}] = l_{21}, & [l_{20},l_{22}] = l_{12}. & \end{array}$$

The procedure of identification of a given contracted Lie algebra $\mathcal{L}$ is described in [9]. Let us briefly review it: at first we compute the **center**

$$C(\mathcal{L}) = \{x \in \mathcal{L};\ \text{for all } y \in \mathcal{L}\,,\ [x,y] = 0\}$$

and the **derived algebra** $D(\mathcal{L}) = [\mathcal{L}, \mathcal{L}]$ of $\mathcal{L}$. If the complement of the derived algebra in the center $X = C(\mathcal{L}) \backslash D(\mathcal{L})$ is non-empty, then the decomposition of $\mathcal{L}$ can be obtained from the decomposition of the quotient algebra $\mathcal{L}/D(\mathcal{L}) = X/D(\mathcal{L}) \oplus \mathcal{L}'/D(\mathcal{L})$ where $D(\mathcal{L}) \subset \mathcal{L}'$. **Centralizer** of the adjoint representation in the ring $R = \mathbb{C}^{n,n}$

$$C_R(\mathrm{ad}(\mathcal{L})) = \{x \in R;\ \text{for all } y \in \mathrm{ad}(\mathcal{L})\,,\ [x, y] = 0\}$$

is determined. Lie algebra $\mathcal{L}$ is decomposable into a direct sum of its ideals if and only if there exists a non-trivial **idempotent** in $C_R(\mathrm{ad}(\mathcal{L}))$, i.e., an element with the property $0 \neq E \neq 1,\ E^2 = E$. In such a case the decomposition has the form

$$\mathcal{L} = \mathcal{L}_0 \oplus \mathcal{L}_1, \qquad [\mathcal{L}_0, \mathcal{L}_1] = 0\,, \qquad [\mathcal{L}_i, \mathcal{L}_i] \subseteq \mathcal{L}_i, \qquad i = 0, 1$$

where $\mathcal{L}_0, \mathcal{L}_1$ are eigen-subspaces of the idempotent E corresponding to the eigenvalues $0, 1$. Next, we compute a **derived sequence** $D^0(\mathcal{L}) = \mathcal{L}$, $D^{k+1}(\mathcal{L}) = [D^k(\mathcal{L}), D^k(\mathcal{L})]$, a **lower central sequence** $(\mathcal{L})^0 = \mathcal{L}$, $(\mathcal{L})^{k+1} = [(\mathcal{L})^k, \mathcal{L}]$, and an **upper central sequence** $C^0(\mathcal{L}) = 0$, $C^{k+1}(\mathcal{L})/C^k(\mathcal{L}) = C(\mathcal{L}/C^k(\mathcal{L}))$. The number of formal invariants (Casimir operators) is given by

$$\tau(\mathcal{L}) = \dim(\mathcal{L}) - \sup_{(x_1,\dots,x_n)} \mathrm{rank}(M_{\mathcal{L}})$$

where $M_{\mathcal{L}}$ is the antisymmetric matrix with entries $(M_{\mathcal{L}})_{ij} = \sum_k c_{ij}^k x_k$, and c_{ij}^k are the structure constants of $\mathcal{L}$. Finally, we determine **radical, nilradical** and **Levi decomposition**.

We will see later that from these "classical" characteristics our algebras $\mathcal{L}, \widetilde{\mathcal{L}}$ cannot be distinguished. Therefore, let us define for $\alpha, \beta, \gamma \in \mathbb{C}$ a subset of $\mathfrak{gl}(\mathcal{L})$

$$\mathcal{A}(\alpha, \beta, \gamma) := \{A \in \mathfrak{gl}(\mathcal{L});\ \alpha A[x, y] = \beta[Ax, y] + \gamma[x, Ay],\ \text{for all } x, y \in \mathcal{L}\}.$$

It is clear that $\mathcal{A}(\alpha, \beta, \gamma)$ is a linear subspace of the vector space $\mathfrak{gl}(\mathcal{L})$. The question for which $\alpha, \beta, \gamma \in \mathbb{C}$ is $\mathcal{A}(\alpha, \beta, \gamma)$ in general a Lie subalgebra of $\mathfrak{gl}(\mathcal{L})$ can be answered analyzing the condition

$$A, B \in \mathcal{A}(\alpha, \beta, \gamma) \Rightarrow AB - BA \in \mathcal{A}(\alpha, \beta, \gamma).$$

One can verify directly the following proposition.

Proposition 1. *If any of the following eight possibilities occurs, the subspace $\mathcal{A}(\alpha, \beta, \gamma) \subset \mathfrak{gl}(\mathcal{L})$ is a Lie subalgebra of $\mathfrak{gl}(\mathcal{L})$:*

1. $\alpha = \beta = \gamma = 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathfrak{gl}(\mathcal{L})$
2. $\alpha = \beta = \gamma \neq 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(1,1,1) =: \mathcal{A}_0$
3. $\alpha = 0, \quad \beta = \gamma \neq 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(0,1,1) =: \mathcal{A}_1$
4. $\alpha = \beta \neq 0, \quad \gamma = 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(1,1,0) =: \mathcal{A}_2$
5. $\alpha = \gamma \neq 0, \quad \beta = 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(1,1,0)$
6. $\alpha = \beta = 0, \quad \gamma \neq 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(0,1,0) =: \mathcal{A}_3$
7. $\alpha = \gamma = 0, \quad \beta \neq 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(0,1,0)$
8. $\alpha \neq 0, \quad \beta = \gamma = 0,$ $\qquad \mathcal{A}(\alpha,\beta,\gamma) = \mathcal{A}(1,0,0) =: \mathcal{A}_4.$

The algebra $\mathcal{A}_0 = \mathrm{Der}(\mathcal{L})$ is the well known Lie algebra of derivations of $\mathcal{L}$, i.e., contains the mappings which satisfy $A[x,y] = [Ax,y] + [x,Ay]$, algebra $\mathcal{A}_2 = C_R(\mathrm{ad}(\mathcal{L}))$ is the centralizer of ad($\mathcal{L}$) in $\mathfrak{gl}(\mathcal{L})$. For algebra $\mathcal{A}_3$

$$\mathcal{A}_3 = \{A \in \mathfrak{gl}(\mathcal{L});\;\; A(\mathcal{L}) \subset C(\mathcal{L})\}$$

holds and thus for its dimension we have $\dim(\mathcal{A}_3) = \dim(\mathcal{L})\dim(C(\mathcal{L}))$. Finally, algebra $\mathcal{A}_4$ is determined by $\mathcal{A}_4 = \{A \in \mathfrak{gl}(\mathcal{L});\;\; A(D(\mathcal{L})) = 0\}$ and thus $\dim(\mathcal{A}_4) = \mathrm{codim}(D(\mathcal{L}))\dim(\mathcal{L})$. We demonstrate the usefulness of the algebra $\mathcal{A}_1$, which consists of operators with the propriety $[Ax,y] = -[x,Ay]$, and the intersection of algebras $\mathcal{A}_0 \cap \mathcal{A}_1$. We adopt the following notation

$$\begin{aligned}\mathrm{inv}(\mathcal{L}) &= (\dim D^k(\mathcal{L}))\;(\dim \mathcal{L}^k)\;(\dim C^k(\mathcal{L}))\;\tau(\mathcal{L}) \\ &= [\dim \mathcal{A}_0, \dim \mathcal{A}_1, \dim \mathcal{A}_2, \dim(\mathcal{A}_0 \cap \mathcal{A}_1)].\end{aligned}$$

Thus we can list the summary of characteristics of our algebras $\widetilde{\mathcal{L}}, \mathcal{L}$ as follows:

$\mathrm{inv}(\mathcal{L})$	$(8,4,0)(8,4,2,0)(2,5,8)$	2	$[16,19,9,11]$
$\mathrm{inv}(\widetilde{\mathcal{L}})$	$(8,4,0)(8,4,2,0)(2,5,8)$	2	$[16,19,9,11]$
$\mathrm{inv}(\mathcal{A}_0)$	$(16,15,6,0)(16,15)(0)$	6	$[16,15,1,6]$
$\mathrm{inv}(\widetilde{\mathcal{A}}_0)$	$(16,15,6,0)(16,15)(0)$	6	$[16,15,1,6]$
$\mathrm{inv}(\mathcal{A}_1)$	$(19,15)(19,15)(0)$	5	$[32,0,1,0]$
$\mathrm{inv}(\widetilde{\mathcal{A}}_1)$	$(19,15)(19,15)(0)$	5	$[32,0,1,0]$
$\mathrm{inv}(\mathcal{A}_2)$	$(9,0)(9,0)(9)$	9	$[81,81,81,81]$
$\mathrm{inv}(\widetilde{\mathcal{A}}_2)$	$(9,0)(9,0)(9)$	9	$[81,81,81,81]$
$\mathrm{inv}(\mathcal{A}_0 \cap \mathcal{A}_1)$	$(11,6,0)(11,6,0)(6,11)$	7	$[43,67,31,31]$
$\mathrm{inv}(\widetilde{\mathcal{A}}_0 \cap \widetilde{\mathcal{A}}_1)$	$(11,4,0)(11,4,0)(7,11)$	7	$[57,78,50,50].$

We observe that these two algebras are both nilpotent and can be properly distinguished as non-isomorphic after computing the structure of Lie algebras $\mathcal{A}_0 \cap \mathcal{A}_1$ and $\widetilde{\mathcal{A}}_0 \cap \widetilde{\mathcal{A}}_1$. Let us mention that for all graded contractions of the Pauli graded $\mathfrak{sl}(3,\mathbb{C})$ we constructed also the **sequences of algebras of derivations** called sometimes "tower series"

$$\mathrm{Der}^k(\mathcal{L}) = \underbrace{\mathrm{Der}(\dots(\mathrm{Der}(\mathcal{L})))}_{k-times}. \tag{7}$$

The result (a sequence of dimensions of $\mathrm{Der}^k(\mathcal{L}), \mathrm{Der}^k(\widetilde{\mathcal{L}})$) is for both cases $16, 16, \dots$ i.e., also does not distinguish $\widetilde{\mathcal{L}}, \mathcal{L}$. We conclude that similar calculations as presented enabled us to distinguish between all graded contractions of the Pauli graded $\mathfrak{sl}(3,\mathbb{C})$ (excluding "parametric" classes of Lie algebras). The numbers of results are presented in the Table 1. Complete results will be published elsewhere. Thus associated Lie algebras $\mathcal{A}_1$, $\mathcal{A}_0 \cap \mathcal{A}_1$ and their structure provide useful enlargement of characteristics of Lie algebras.

Dimension of non-Abelian part	Solvable		Nilpotent		Total
	Indecomp.	Decomp.	Indecomp.	Decomp.	
3			1		1
4	1		1		2
5	1		4		5
6	1		9	1	11
7	4	1	28	1	34
8	11	2	77	3	93

Table 1. The numbers of contracted Lie algebras from Pauli graded $\mathfrak{sl}(3,\mathbb{C})$.

4. Concluding Remarks

Let us mention further several interesting issues.

- If we define a Lie group of regular linear mappings $U \in \mathrm{GL}(\mathcal{L})$ satisfying $[Ux, Uy] = [x, y]$ for all $x, y \in \mathcal{L}$ then $\mathcal{A}_1$ is a Lie algebra of this Lie group.
- The following relation

$$[\mathcal{A}_0, \mathcal{A}_i] \subset \mathcal{A}_i, \quad i = 1, 2, 3, 4$$

also holds. Moreover, $\mathcal{A}_0 \cap \mathcal{A}_1$ is an ideal in $\mathcal{A}_0$.

- Similarly to the tower series (7) of algebras of derivations we may investigate such analogous series of algebras $\mathcal{A}_1$ and $\mathcal{A}_0 \cap \mathcal{A}_1$. Moreover, we may combine these series, e.g., we may compute algebra $\mathcal{A}_1$ for $\mathrm{Der}(\mathcal{L})$ and so on. This raises the question of dependence relations between such obtained algebras.

Classification of contractions or graded contractions of $\mathfrak{sl}(3,\mathbb{C})$ is complicated due to missing classification of eight-dimensional Lie algebras. These eight-dimensional Lie algebras are still very useful in physics, geometry, theory of PDE and their complete classification would lead immediately to new results in those fields. Associated Lie algebras $\mathcal{A}_1$, $\mathcal{A}_0 \cap \mathcal{A}_1$ and their series may partially supply the “lack of invariants” which causes such classification out of reach.

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Note added in proof: The concept of associated Lie algebras is a special case of so called generalized derivations, for general setting see e.g. [6].

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