TOKYO J. MATH.
VOL. 7, NO. 2, 1984

A Supplement to the Paper "Non-compact and Non-trivial Minimal Sets of a Locally Compact Flow"

Shigeo KONO

Josai University

(Communicated by T. Saito)

Introduction

We determined the structure of the non-compact and non-trivial minimal sets of a locally compact flow in the paper [1]. The aim of the present paper is to supplement the paper mentioned above, by giving the structure of the non-compact and trivial minimal set of a locally compact flow.

As references for notations and definitions used here, consult [1].

§1. Non-compact and trivial minimal sets of a locally compact flow.

A minimal set is called trivial, if it consists of only one trajectory.

LEMMA 1. *Let (X, R, π) be a locally compact flow. Then the trajectory $C(x)$ of (X, R, π) is periodic if and only if*

$$L^+(x)=C(x) \quad or \quad L^-(x)=C(x)$$

holds.

PROOF. If $C(x)$ is periodic, then it is clear that $L^+(x)=L^-(x)=C(x)$. Now assume that $L^+(x)=C(x)$. Then $C(x)$ is positively Poisson stable. It is known that if (Y, R, g) is a locally compact flow and the trajectory $C(y)$ of (Y, R, g) is positively Poisson stable but non-periodic, then

$$\overline{L^+(y)-C(y)}=\overline{C(y)}$$

holds [2, p. 60, 4.6]. By virtue of this fact we can show that $C(x)$ is periodic. For, if $C(x)$ is not periodic, then we have

$$\overline{L^+(x)-C(x)}=\overline{C(x)} ,$$

which implies that $C(x)$ is empty, since $L^+(x)=C(x)$ by the assumption.

Received August 1, 1984.

But this is absurd. The proof is similar in case where $L^-(x)=C(x)$.
Q.E.D.

By virtue of Lemma 1 we obtain the following result.

THEOREM 1. *Every non-compact and trivial minimal set of a locally compact flow consists of a receding trajectory.*

PROOF. Let (X, R, π) be a locally compact flow. M is assumed to be a non-compact and trivial minimal set of (X, R, π). Then M consists of a single trajectory, say $C(x)$, such that $M=C(x)$. On the other hand we have

$$L^+(x)=M \quad \text{or} \quad L^+(x)=\varnothing$$

and

$$L^-(x)=M \quad \text{or} \quad L^-(x)=\varnothing ,$$

since M is minimal and both $L^+(x)$ and $L^-(x)$ are closed and invariant. Hence four cases are possible:

1° $L^+(x)=L^-(x)=M$,
2° $L^+(x)=M$ and $L^-(x)=\varnothing$,
3° $L^+(x)=\varnothing$ and $L^-(x)=M$,
4° $L^+(x)=L^-(x)=\varnothing$.

Assume that the case 1° holds. Then we have $L^+(x)=C(x)=M$, so that $C(x)$ is periodic by virtue of Lemma 1, and hence $C(x)$ must be compact. This, however, contradicts the non-compactness of the set M. Next assume that the case 2° holds. Then we have $L^+(x)=C(x)$, which implies that $C(x)$ is periodic. In this case $L^-(x)$ is not empty, since $C(x)$ is compact. This is again a contradiction. Similarly we can show that the case 3° does not hold. Thus we conclude that only the case 4° is valid.
Q.E.D.

References

[1] S. KONO, Non-compact and non-trivial minimal sets of a locally compact flow, Tokyo J. Math., 5 (1982), 213–223.

[2] N. P. BHATIA and O. HAJEK, Theory of Dynamical Systems, Part I, Technical Note BN-599, University of Maryland, 1969.

Present Address:
DEPARTMENT OF MATHEMATICS
JOSAI UNIVERSITY
1-1, KEYAKI-DAI,
SAKADO, 350-02