On log canonical rational singularities

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Abstract: We prove that the class of log canonical rational singularities is closed under the basic operations of the minimal model program. We also give some supplementary results on the minimal model program for log canonical surfaces.

Key words: Rational singularities; log canonical singularities; minimal model program; log canonical surfaces.

1. Introduction. In this short note, we prove the following theorems, which are missing in [3]. This short note is a supplement to [3], [6], and [4].

Theorem 1.1. *Let (X, Δ) be a log canonical pair and let $f : X \to Y$ be a projective surjective morphism such that $f_*\mathcal{O}_X \simeq \mathcal{O}_Y$ and that $-(K_X + \Delta)$ is f-ample. Assume that X has only rational singularities. Then Y has only rational singularities.*

We can easily prove Theorem 1.1 by the relative Kodaira type vanishing theorem for log canonical pairs and Kovács's characterization of rational singularities. Of course, the vanishing theorem for log canonical pairs is nontrivial in the classical minimal model program (see [10]). However, now we can freely use such a powerful vanishing theorem for log canonical pairs (see, for example, [3] and [5]). Note that we do not assume that f is birational in Theorem 1.1.

Theorem 1.2. *We consider a commutative diagram*

$$\begin{array}{ccc} X & \overset{\phi}{\dashrightarrow} & X^+ \\ & {\scriptstyle f}\searrow \quad \swarrow{\scriptstyle f^+} & \\ & Y & \end{array}$$

where (X, Δ) and (X^+, Δ^+) are log canonical, f and f^+ are projective birational morphisms, and Y is normal. Assume that

(i) $f_*\Delta = f^+_*\Delta^+$,

(ii) *$-(K_X + \Delta)$ is f-ample, and*

(iii) *$K_{X^+} + \Delta^+$ is f^+-ample.*

We further assume that X has only rational singularities. Then X^+ has only rational singularities.

Theorem 1.2 follows from the well-known negativity lemma (see, for example, [10, Lemma 3.38] and [5, Lemma 2.3.27]) and the result on nonrational centers of log canonical pairs due to Alexeev–Hacon (see [1]), which can be obtained in the framework of [3].

Remark 1.3. In Theorem 1.2, the log canonicity of (X^+, Δ^+) follows from the other conditions of Theorem 1.2 by the negativity lemma (see, for example, [10, Lemma 3.38] and [5, Lemma 2.3.27]). It is sufficient to assume that X^+ is a normal variety and Δ^+ is an effective $\mathbf{R}$-divisor on X^+ such that $K_{X^+} + \Delta^+$ is $\mathbf{R}$-Cartier.

Note that the singularities of X are not always rational when (X, Δ) is only log canonical. Moreover, X is not necessarily Cohen–Macaulay. This is one of the difficulties when we treat log canonical pairs. We hope that Theorem 1.1 and Theorem 1.2 will be useful for the study of log canonical pairs.

1.4 (MMP for log canonical pairs with only rational singularities)**.** Let us discuss the minimal model program for log canonical pairs with only rational singularities.

Let (X, Δ) be a log canonical pair and let $\pi : X \to S$ be a projective morphism onto a variety S. Then we know that we can always run the minimal model program starting from $\pi : (X, \Delta) \to S$ (for the details, see, for example, [3], [2], [9], [6], [5], and so on). We further assume that X has only rational singularities. Then, Theorem 1.1 and Theorem 1.2 say that every variety appearing in the minimal model program starting from $\pi : (X, \Delta) \to S$ has only rational singularities.

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From now on, we will study a contraction morphism more precisely. Let

$$f : (X, \Delta) \to Y$$

be a contraction morphism such that

(i) (X, Δ) is a $\mathbf{Q}$-factorial log canonical pair,
(ii) $-(K_X + \Delta)$ is f-ample, and
(iii) $\rho(X/Y) = 1$.

Then we have the following three cases.

Case 1 (Divisorial contraction). f is divisorial, that is, f is a birational contraction which contracts a divisor. In this case, the exceptional locus $\mathrm{Exc}(f)$ of f is a prime divisor on X and (Y, Δ_Y) is a $\mathbf{Q}$-factorial log canonical pair with $\Delta_Y = f_*\Delta$. Moreover, if X has only rational singularities, then Y has only rational singularities by Theorem 1.1.

Case 2 (Flipping contraction). f is flipping, that is, f is a birational contraction which is small. In this case, we can take the flipping diagram:

$$\begin{array}{ccc} X & \overset{\varphi}{\dashrightarrow} & X^+ \\ & {\scriptstyle f}\searrow \quad \swarrow{\scriptstyle f^+} & \\ & Y & \end{array}$$

where f^+ is a small projective birational morphism and

(i′) (X^+, Δ^+) is a $\mathbf{Q}$-factorial log canonical pair with $\Delta^+ = \varphi_*\Delta$,
(ii′) $K_{X^+} + \Delta^+$ is f^+-ample, and
(iii′) $\rho(X^+/Y) = 1$.

By Theorem 1.2, we see that X^+ has only rational singularities when X has only rational singularities. For the existence of log canonical flips, see [2, Corollary 1.2] and [9, Corollary 1.8].

Case 3 (Fano contraction). f is a Fano contraction, that is, $\dim Y < \dim X$. Then Y is $\mathbf{Q}$-factorial and has only log canonical singularities by [6]. Moreover, if X has only rational singularities, then Y has only rational singularities by Theorem 1.1.

Therefore, the class of $\mathbf{Q}$-factorial log canonical rational singularities is closed under the minimal model program.

Let (X, Δ) be a projective log canonical pair such that $K_X + \Delta$ is a semiample big $\mathbf{Q}$-Cartier divisor. Unfortunately, the log canonical model of (X, Δ) may have nonrational singularities even when X has only rational singularities (see Example 5.1). This causes some undesirable phenomena (see Example 5.3).

In this paper, we also give some supplementary results on the minimal model program for (not necessarily $\mathbf{Q}$-factorial) log canonical surfaces. We have:

Theorem 1.5 (see Theorem 4.1). *Let (X, Δ) be a log canonical surface and let $f : X \to Y$ be a projective birational morphism onto a normal surface Y. Assume that $-(K_X + \Delta)$ is f-ample. Then the exceptional locus* $\mathrm{Exc}(f)$ *of f passes through no nonrational singular points of X.*

By Theorem 1.5, the minimal model program for log canonical surfaces discussed in [4, Theorem 3.3] becomes independent of the classification of numerically lc surface singularities in [10, Theorem 4.7] (see Remark 4.4). When a considered surface is not $\mathbf{Q}$-factorial, the original proof of [4, Theorem 3.3] uses the fact that a numerically lc surface is a log canonical surface (see [4, Proposition 3.5 (2)]). For the proof of this fact, we need a rough classification of numerically lc surface singularities in [10, Theorem 4.7] (see the proof of [4, Proposition 3.5 (2)]).

We will work over $\mathbf{C}$, the complex number field, throughout this short note. We will freely use the basic notation of the minimal model program as in [3].

2. Preliminaries.

Let us recall the notion of singularities of pairs. For the details, see [3], [5], and so on.

2.1 (Singularities of pairs). A pair (X, Δ) consists of a normal variety X and an effective $\mathbf{R}$-divisor Δ on X such that $K_X + \Delta$ is $\mathbf{R}$-Cartier. A pair (X, Δ) is called kawamata log terminal (resp. log canonical) if for any projective birational morphism $f : Y \to X$ from a normal variety Y, $a(E, X, \Delta) > -1$ (resp. ≥ -1) for every E, where

$$K_Y = f^*(K_X + \Delta) + \sum_E a(E, X, \Delta)E.$$

Let (X, Δ) be a log canonical pair and let W be a closed subset of X. Then W is called a log canonical center of (X, Δ) if there are a projective birational morphism $f : Y \to X$ from a normal variety Y and a prime divisor E on Y such that $a(E, X, \Delta) = -1$ and that $f(E) = W$. Let (X, Δ) be a log canonical pair. If there exists a projective birational morphism $f : Y \to X$ from a smooth variety Y such that the f-exceptional locus $\mathrm{Exc}(f)$ and $\mathrm{Exc}(f) \cup \mathrm{Supp}\, f_*^{-1}\Delta$ are simple normal crossing divisors on Y and that $a(E, X, \Delta) > -1$ for every f-exceptional divisor E, then (X,Δ) is called a divisorial log terminal pair.

For surfaces, we can define $a(E,X,\Delta)$ without assuming that $K_X+\Delta$ is $\mathbf{R}$-Cartier. Then we can define numerically lc surfaces and numerically dlt surfaces (see [10, Notation 4.1]). Precisely speaking, we have:

2.2 (Numerically lc and dlt due to Kollár–Mori (see [10, Notation 4.1]))**.** Let X be a normal surface and let Δ be an $\mathbf{R}$-divisor on X whose coefficients are in $[0,1]$. Let $f:Y\to X$ be a projective birational morphism from a smooth variety Y with the exceptional divisor $E=\sum_i E_i$. Then the system of linear equations

$$E_j\cdot\left(\sum_i a_iE_i\right)=E_j\cdot(K_Y+f_*^{-1}\Delta)$$

for any j has a unique solution. We write this as

$$K_Y+f_*^{-1}\Delta\equiv\sum_i a(E_i,X,\Delta)E_i$$

with $a(E_i,X,\Delta)=a_i$. In this situation, we say that (X,Δ) is numerically lc if $a(E_i,X,\Delta)\geq -1$ for every exceptional curve E_i and every resolution of singularities $f:Y\to X$. We say that (X,Δ) is numerically dlt if there exists a finite set $Z\subset X$ such that $X\setminus Z$ is smooth, $\operatorname{Supp}\Delta|_{X\setminus Z}$ is a simple normal crossing divisor, and $a(E,X,\Delta)>-1$ for every exceptional curve E which maps to Z.

Let us recall the basic operations and notation for $\mathbf{R}$-divisors.

2.3 ($\mathbf{R}$-divisors)**.** Let $D=\sum a_iD_i$ be an $\mathbf{R}$-divisor on a normal variety X. Note that D_i is a prime divisor for every i and that $D_i\neq D_j$ for $i\neq j$. Of course, $a_i\in\mathbf{R}$ for every i. We put $\lfloor D\rfloor=\sum\lfloor a_i\rfloor D_i$ and call it the round-down of D. Note that, for every real number x, $\lfloor x\rfloor$ is the integer defined by $x-1<\lfloor x\rfloor\leq x$. We also put $\lceil D\rceil=-\lfloor -D\rfloor$ and call it the round-up of D. The fractional part $\{D\}$ denotes $D-\lfloor D\rfloor$. We put

$$D^{=1}=\sum_{a_i=1}D_i \quad\text{and}\quad D^{<1}=\sum_{a_i<1}a_iD_i.$$

Let B_1 and B_2 be two $\mathbf{R}$-Cartier divisors on a normal variety X. Then B_1 is $\mathbf{R}$-linearly equivalent to B_2, denoted by $B_1\sim_{\mathbf{R}}B_2$, if

$$B_1=B_2+\sum_{i=1}^k r_i(f_i)$$

such that $f_i\in\mathbf{C}(X)$ and $r_i\in\mathbf{R}$ for every i. We note that (f_i) is a principal Cartier divisor associated to f_i. Let $f:X\to Y$ be a morphism to a variety Y. If there is an $\mathbf{R}$-Cartier divisor B on Y such that

$$B_1\sim_{\mathbf{R}}B_2+f^*B,$$

then B_1 is said to be relatively $\mathbf{R}$-linearly equivalent to B_2. It is denoted by $B_1\sim_{\mathbf{R},f}B_2$ or $B_1\sim_{\mathbf{R},Y}B_2$.

3. Proof of theorems. In this section, we prove Theorem 1.1 and Theorem 1.2. Let us prove Theorem 1.1.

Proof of Theorem 1.1. By Kodaira type vanishing theorem for log canonical pairs (see, for example, [3, Theorem 8.1] and [5, Theorem 5.6.4]), we have $R^if_*\mathcal{O}_X=0$ for every $i>0$. Therefore, we have $Rf_*\mathcal{O}_X\simeq\mathcal{O}_Y$. Then, by Kovács's characterization of rational singularities (see [11, Theorem 1] and [5, Theorem 3.12.5]), we obtain that Y has only rational singularities. When f is birational, see also Lemma 3.1 below. □

The following lemma is obvious by the definition of rational singularities.

Lemma 3.1. *Let $f:X\to Y$ be a proper birational morphism between normal varieties. Assume that $R^if_*\mathcal{O}_X=0$ for every $i>0$. Then X has only rational singularities if and only if Y has only rational singularities.*

Here, we give a proof of [1, Theorem 1.2], which is a main ingredient of Theorem 1.2, for the reader's convenience.

Theorem 3.2 ([1, Theorem 1.2])**.** *Let (X,Δ) be a log canonical pair and let $f:Y\to X$ be a resolution of singularities. Then every associated prime of $R^if_*\mathcal{O}_Y$ is the generic point of some log canonical center of (X,Δ) for every $i>0$.*

Note that $R^if_*\mathcal{O}_Y$ is independent of the resolution $f:Y\to X$.

Proof. Without loss of generality, we may assume that X is quasi-projective by shrinking X. We take a dlt blow-up $g:(Z,\Delta_Z)\to(X,\Delta)$ (see, for example, [5, Theorem 4.4.21] and [3, Section 10]). This means that g is a projective birational morphism such that $K_Z+\Delta_Z=g^*(K_X+\Delta)$ and that (Z,Δ_Z) is a divisorial log terminal pair. It is well known that Z has only rational singularities. We take a projective birational morphism $h:Y\to Z$ such that $K_Y+\Delta_Y=h^*(K_Z+\Delta_Z)$, Y is smooth, and $\operatorname{Supp}\Delta_Y$ is a simple normal crossing divisor on Y. We may assume that h is an isomorphism over the generic point of any log canonical center of (Z,Δ_Z) by Szabó's resolution lemma (see, for example, [5, Remark 2.3.18 and Lemma 2.3.19]). Then we have

$$K_Y + \{\Delta_Y\} + \Delta_Y^{=1} + \lfloor \Delta_Y^{<1} \rfloor = K_Y + \Delta_Y \sim_{\mathbf{R},f} 0,$$

where $f = g \circ h : Y \to X$. We put $E = \lceil -\Delta_Y^{<1} \rceil$. Then E is effective, h-exceptional, and $E \sim_{\mathbf{R},f} K_Y + \{\Delta_Y\} + \Delta_Y^{=1}$. Therefore, we obtain $Rh_*\mathcal{O}_Y(E) \simeq \mathcal{O}_Z$ since $R^i h_* \mathcal{O}_Y(E) = 0$ for every $i > 0$ by the vanishing theorem of Reid–Fukuda type (see, for example, [3, Lemma 6.2] and [5, Theorem 3.2.11]) and $h_*\mathcal{O}_Y(E) \simeq \mathcal{O}_Z$. Note that $Rh_*\mathcal{O}_Y \simeq \mathcal{O}_Z$ since Z has only rational singularities. Thus, we obtain

$$\begin{aligned} Rf_*\mathcal{O}_Y(E) &\simeq Rg_*Rh_*\mathcal{O}_Y(E) \simeq Rg_*\mathcal{O}_Z \\ &\simeq Rg_*Rh_*\mathcal{O}_Y \simeq Rf_*\mathcal{O}_Y. \end{aligned}$$

By [3, Theorem 6.3 (i)] (see also [5, Theorem 3.16.3 (i)]), we have that every associated prime of $R^i f_*\mathcal{O}_Y(E) \simeq R^i f_*\mathcal{O}_Y$ is the generic point of some log canonical center of (X, Δ) for every $i > 0$. □

Let us prove Theorem 1.2.

Proof of Theorem 1.2. Let $g : Z \to X^+$ be a resolution of singularities. Let $\mathrm{Exc}(f^+)$ be the exceptional locus of $f^+ : X^+ \to Y$. By Theorem 1.1, we know that Y has only rational singularities. Therefore, $X^+ \setminus \mathrm{Exc}(f^+)$ has only rational singularities. Thus, $\mathrm{Supp}\, R^i g_*\mathcal{O}_Z \subset \mathrm{Exc}(f^+)$ for every $i > 0$. By the negativity lemma (see, for example, [10, Lemma 3.38] and [5, Lemma 2.3.27]), there are no log canonical centers of (X^+, Δ^+) contained in $\mathrm{Exc}(f^+)$. By Theorem 3.2, every associated prime of $R^i g_*\mathcal{O}_Z$ is the generic point of some log canonical center of (X^+, Δ^+) for every $i > 0$. Thus, we have $R^i g_*\mathcal{O}_Z = 0$ for every $i > 0$. This means that X^+ has only rational singularities. □

4. On log surfaces. In this section, we give some results on the minimal model program for log canonical surfaces (see [4], [8], and [13]). This section is a supplement to [4].

The following theorem is the main result of this section.

Theorem 4.1. *Let (X, Δ) be a log canonical surface and let $f : X \to Y$ be a projective birational morphism onto a normal surface Y. Assume that $-(K_X + \Delta)$ is f-ample. Then the exceptional locus* $\mathrm{Exc}(f)$ *of f passes through no nonrational singular points of X. In particular, every f-exceptional curve is a* $\mathbf{Q}$*-Cartier divisor. Moreover, if the relative Picard number $\rho(X/Y) = 1$, then* $\mathrm{Exc}(f)$ *is an irreducible curve and $K_Y + \Delta_Y$, where $\Delta_Y = f_*\Delta$, is* $\mathbf{R}$*-Cartier.*

Proof. By shrinking Y, we may assume that $f(\mathrm{Exc}(f)) = P$ and that (Y, Δ_Y), where $\Delta_Y = f_*\Delta$, is numerically dlt by the negativity lemma (see, for example, [10, Lemma 3.41] and [5, Lemma 2.3.25]). Therefore, Y has only rational singularities (see [10, Theorem 4.12]). By the Kodaira type vanishing theorem as in the proof of Theorem 1.1 (see also [8, Theorem 6.2]), we obtain $R^i f_*\mathcal{O}_X = 0$ for every $i > 0$. Thus, X has only rational singularities in a neighborhood of $\mathrm{Exc}(f)$ by Lemma 3.1. This means that X is $\mathbf{Q}$-factorial around $\mathrm{Exc}(f)$ (see, for example, [12, Proposition (17.1)] and [13, Proposition 20.2]). Therefore, every f-exceptional curve is a $\mathbf{Q}$-Cartier divisor. From now on, we assume that $\rho(X/Y) = 1$. We take an irreducible f-exceptional curve E. Then $E^2 < 0$ and $E \cdot C < 0$ for every f-exceptional curve C. This means that $E = \mathrm{Exc}(f)$. We can take a real number a such that $(K_X + \Delta + aE) \cdot E = 0$. Then, by the contraction theorem (see [4, Theorem 3.2] and [13, Theorem 17.1]), we can check that $K_Y + \Delta_Y$ is $\mathbf{R}$-Cartier and $K_X + \Delta + aE = f^*(K_Y + \Delta_Y)$. □

As an easy consequence of Theorem 4.1, we have:

Corollary 4.2. *In the minimal model program for log canonical surfaces, the number of nonrational log canonical singularities never decreases.*

Remark 4.3. Theorem 4.1 and Corollary 4.2 hold true over any algebraically closed field k. This is because the vanishing theorems for birational morphisms from log surfaces hold true even when the characteristic of k is positive (see, for example, [8, Theorem 6.2]).

We give an important remark on [4].

Remark 4.4. In [4], we used the fact that a numerically lc surface is a log canonical surface (see [4, Proposition 3.5 (2)]) for the proof of the minimal model program for (not necessarily $\mathbf{Q}$-factorial) log canonical surfaces (see [4, Theorem 3.3]). Note that the proof of [4, Proposition 3.5 (2)] more or less depends on the classification of numerically lc surface singularities in [10, Theorem 4.7]. By using Theorem 4.1, we can check that $K_{X_i} + \Delta_i$ is $\mathbf{R}$-Cartier in the proof of [4, Theorem 3.3] without using [4, Proposition 3.5 (2)]. This means that the minimal model program for log canonical surfaces in [4, Theorem 3.3] is independent of the classification of (numerically) lc surface singularities (see [10, Theorem 4.7]).

5. Examples. In this section, we will see that nonrational singularities sometimes may cause undesirable phenomena.

Note that the log canonical model of a log canonical surface may have nonrational singularities.

Example 5.1. Let $C \subset \mathbf{P}^2$ be an elliptic curve and let $V \subset \mathbf{P}^3$ be a cone over $C \subset \mathbf{P}^2$. Let $p : X \to V$ be the blow-up at the vertex P of V. Note that P is an isolated log canonical Gorenstein singularity and p is the minimal resolution. We take a general very ample smooth Cartier divisor Δ_V on V such that $K_V + \Delta_V$ is very ample. We put $K_X + \Delta = p^*(K_V + \Delta_V)$. Then X is smooth, (X, Δ) is log canonical, and $K_X + \Delta$ is big. Note that $p = \Phi_{|K_X+\Delta|} : X \to V$ by construction. We also note that (V, Δ_V) is log canonical and that the singularity $P \in V$ is not rational. Of course, (V, Δ_V) is the log canonical model of (X, Δ).

A finite étale morphism between kawamata log terminal pairs of log general type induces a natural finite étale cover of their log canonical models in any dimension.

Theorem 5.2. *Let X be a normal projective variety and let Δ be an effective $\mathbf{Q}$-divisor on X such that (X, Δ) is kawamata log terminal. Let $f : Y \to X$ be a finite étale morphism such that $K_Y + \Delta_Y = f^*(K_X + \Delta)$. Assume that $K_X + \Delta$ is big. Then we have a commutative diagram*

$$\begin{array}{ccc} Y & \overset{q}{\dashrightarrow} & Y_c \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f_c} \\ X & \underset{p}{\dashrightarrow} & X_c \end{array}$$

where p and q are birational maps, (X_c, Δ_c) (resp. (Y_c, Δ_{Y_c})) is the log canonical model of (X, Δ) (resp. (Y, Δ_Y)), f_c is a finite étale morphism, and $K_{Y_c} + \Delta_{Y_c} = f_c^*(K_{X_c} + \Delta_c)$.

Proof. The proof of [7, Theorem 4.5] works with some suitable modifications. Note that X_c and Y_c have only rational singularities since (X_c, Δ_c) and (Y_c, Δ_{Y_c}) are both kawamata log terminal pairs. We leave the details as an exercise for the reader. □

Unfortunately, Theorem 5.2 does not hold for log canonical pairs. This is because log canonical models of log canonical pairs sometimes have nonrational singularities.

Example 5.3. Let $p : X \to V$ be as in Example 5.1 and let E be the p-exceptional divisor on X. Note that there is a natural $\mathbf{P}^1$-bundle structure $\pi : X \to C$ and E is a section of π. We take a nontrivial finite étale cover $D \to C$. We put $Y = X \times_C D$ and $F = E \times_C D$. We put $K_Y + \Delta_Y = f^*(K_X + \Delta)$, where $f : Y \to X$ is the natural induced étale morphism. Let W be the log canonical model of (Y, Δ_Y). Then we have the following commutative diagram

$$\begin{array}{ccc} Y & \xrightarrow{q} & W \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle h} \\ X & \xrightarrow[p]{} & V \end{array}$$

such that f is étale and h is finite. Note that q contracts F to an isolated normal singular point Q of W such that $h^{-1}(P) = Q$ since $f^{-1}(E) = F$. Therefore, h is not étale by $\deg h > 1$ and $h^{-1}(P) = Q$. We also note that the singularities of V and W are not rational since E and F are elliptic curves. This example says that Theorem 5.2 does not always hold for log canonical pairs.

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