Corestriction principle for non-abelian cohomology of reductive group schemes over arithmetical rings

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Abstract: We prove some new results on Corestriction principle for non-abelian cohomology of group schemes over local and global fields or the rings of integers thereof.

Key words: Corestriction principle; norm principle; group schemes; arithmetical rings.

1. Introduction. In [T1–T3] we proved some results on Corestriction principle for connecting maps of non-abelian Galois cohomology of reductive groups over local and global fields. In [T3] there was defined also a concept of Weak Corestriction principle for non-abelian Galois cohomology of such groups over arbitrary fields of charateristic 0. It is apparent and natural to consider similar notions for *groups of arithmetical types*, i.e., consider group schemes over arithmetical rings, which are either local or global fields, or their ring of integers. Such a treatment over rings is necessary for various arithmetic considerations. For example, in [X], there has been proved the validity of Corestriction principle, under some restrictions, for spinor norms over the ring of $\mathfrak{p}$-adic integers.

We consider in this paper the concept of Corestriction principle (resp. Weak Corestriction principle) in a setting, more general than that of Galois cohomology. The full proofs of results presented here will appear elsewhere. The definitions are similar so we only briefly recall it below (see [T1–T3] for more details). For arithmetical applications, we consider only the case of Dedekind rings (or their localizations or completions with respect to discrete valuations) and their quotient fields. We call such rings in this paper by *arithmetical rings*. For such a ring A and a group scheme G over A (i.e. over $Spec(A)$), we denote as usual $\mathrm{H}^i_r(A,G) := \mathrm{H}^i_r(Spec(A),G)$, where r stands either for Zariski, étale, or flat (i.e., fppf) topology, whenever it makes sense. We assume once for all that, for all smooth *commutative* A-group schemes involved, there is a notion of corestriction homomorphism, that is, for any smooth commutative A-group scheme T and each extension A'/A (and also their localizations at finite sets of primes) belonging to certain category $\mathcal{C}_{\mathcal{A}}$ of faithfully flat, étale extensions of finite type over A, there is a functorial homomorphism

$$Cores_{A'/A,T} : \mathrm{H}^i_{et}(A', T_{A'}) \to \mathrm{H}^i_{et}(A,T).$$

Here $T_{A'} = T \times_A A'$ denotes the A'-group scheme obtained by base change from A to A'. In general, one may not expect such homomorphism to exist and there is a general theory of trace handling this question in Deligne [SGA 4], Exp. 17 (cf. also Gille [Gi]). One may then consider the concept of (Weak) Corestriction principle for images or kernels of connecting maps in a long exact sequence of cohomology.

Assume that we have a map which is functorial in B, $B \in \mathcal{C}_A$:

$$\alpha_B : \mathrm{H}^p_{et}(B,G_B) \to \mathrm{H}^q_{et}(B,T_B),$$

i. e., a map of functors $\alpha = (\alpha_B) : (B \mapsto \mathrm{H}^p_{et}(B,G_B)) \to (B \mapsto \mathrm{H}^q_{et}(B,T_B))$ where B runs over all $\mathcal{C}_A$, T is a commutative algebraic A-group scheme. It is natural to ask whether or not the following inclusion holds

$$Cores_{B/A,T}(\mathrm{Im}\ (\alpha_B)) \subset \mathrm{Im}\ (\alpha_A).$$

If it is the case for $B \in \mathcal{C}_A$, then we say that the Corestriction principle holds for the image of the map $\alpha_A : \mathrm{H}^p_{et}(A,G) \to \mathrm{H}^q_{et}(A,T)$ with respect to $Cores_{B/A,T}$. We say that Weak Corestriction principle holds for the image of α_A with respect to $Cores_{B/A,T}$, if

$$Cores_{B/A,T}(\mathrm{Im}\ (\alpha_B)) \subset \langle \mathrm{Im}\ (\alpha_A)\rangle,$$

where $\langle Im(\alpha_A)\rangle$ denotes the subgroup generated in the cohomology group by Im (α_A). We may also

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consider similar notions for kernel of α_A, when G is commutative and T may be not.

2. Main Results. In this paper we prove the following analogs of the results proved in the case of local and global fields.

Theorem 1 (Local Corestriction principle). *Let A be a ring of integers of a non-archimedean local field k, A' is the integral closure of A in a separable finite extension k' of k, belonging to $\mathcal{C}_{\mathcal{A}}$. Let G, T be reductive A-group schemes with T an A-torus, and let $\alpha_A : \mathrm{H}^p_{et}(A, G) \to \mathrm{H}^q_{et}(A, T)$, (resp. $\alpha_A : \mathrm{H}^1_{et}(A, T) \to \mathrm{H}^1_{et}(A, G)$) be a connecting map induced by exact sequences of A-group schemes involving G and T (resp. induced by A-morphism $T \to G$). Then Corestriction Principle holds for the image (resp. kernel) of α_A with respect to $Cores_{A'/A,T}$.*

Examples. 1) Let $\mathcal{O}_v$ be the ring of integers of a local non-archimedean field k, G, T reductive $\mathcal{O}_v$-group schemes, where T is a torus, and let $\pi : G \to T$ be a morphism of $\mathcal{O}_v$-group schemes. For any finite separable unramified extension k'/k with the ring of integers $\mathcal{O}_w$ there is a natural norm homomorphism

$$N := N_{\mathcal{O}_w/\mathcal{O}_v} : T(\mathcal{O}_w) \to T(\mathcal{O}_v),$$

and in the following diagram

$$\begin{array}{ccc} G(\mathcal{O}_w) & \xrightarrow{\beta'} & T(\mathcal{O}_w) \\ & & \downarrow N \\ G(\mathcal{O}_v) & \xrightarrow{\beta} & T(\mathcal{O}_v) \end{array}$$

we have

$$N(\beta'(G(\mathcal{O}_w))) \subset \beta(G(\mathcal{O}_v)).$$

2) Let $\alpha : T \to G$ be an A-homomorphism as in Theorem 1, $K = \mathrm{Ker}\,(\alpha)$, $T' = \mathrm{Im}\,(\alpha)$. Then α induces exact sequences $1 \to K \to T \to T' \to 1$, and $1 \to T' \to G \to G/T' \to 1$, and therefore long exact sequences of cohomologies. The induced map $\alpha_A : \mathrm{H}^1_{et}(A, T) \to \mathrm{H}^1_{et}(A, G)$ is the composition of $\mathrm{H}^1_{et}(A, T) \to \mathrm{H}^1_{et}A, T')$ and $\mathrm{H}^1_{et}(A, T') \to \mathrm{H}^1_{et}(A, G)$. Thus the study of Ker (α_A) is reduced to that of $\mathrm{H}^1_{et}(A, T') \to \mathrm{H}^1_{et}(A, G)$, and we may assume that T is an A-subtorus of G. Then Ker $(\mathrm{H}^1_{et}(A, T) \to \mathrm{H}^1_{et}(A, G)) = \mathrm{Im}\,(\mathrm{H}^0_{et}(A, (G/T)) \to \mathrm{H}^1_{et}(A, T))$. Here $\mathrm{H}^0_{et}(A, (G/T)) = (G/T)(A)$ may not be a group.

In the global case, we have a similar (but less satisfactory) results as follows:

Theorem 2 (Global Corestriction principle). *Let A be the ring of integers[1)] of a global field k, $\alpha_A : \mathrm{H}^p_{et}(A, G) \to \mathrm{H}^q_{et}(A, T)$ (resp. $\alpha_A : \mathrm{H}^1_{et}(A, T) \to \mathrm{H}^1_{et}(A, G)$) a connecting map induced by an exact sequence of cohomologies of reductive A-group schemes (resp. an A-morphism), with T an A-torus. Then for any finite separable extension k'/k with the ring of integers A' belonging to $\mathcal{C}_A$, there is a finite set $S \subset V$ such that the Corestriction Principle holds for the image (resp. kernel) of α_{A_S} : $\mathrm{H}^p_{et}(A_S, G_{A_S}) \to \mathrm{H}^q_{et}(A_S, T_{A_S})$ (resp. $\alpha_{A_S} : \mathrm{H}^1_{et}(A_S, T_{A_S}) \to \mathrm{H}^1_{et}(A_S, G_{A_S})$) with respect to $Cores_{A'_S/A_S,T}$.*

While in the case of étale cohomology it is possible to define the corestriction maps due to an analog of Shapiro lemma (cf. [SGA 3], Exp. XXIV, Sec. 8, Prop. 8.4), it is not in general the case if we consider the case of flat cohomology. However, if the base scheme is the spectrum of a local or global field then we can prove the Corestriction principle for algebraic groups and we have the following

Theorem 3. *Let k be a local or global field of characteristic > 0.*
a) Let $\alpha_k : \mathrm{H}^p_{fppf}(k, G) \to \mathrm{H}^q_{fppf}(k, T)$ be a connecting map induced by an exact sequence involving k-groups. Assume that G is connected, reductive and T is a torus. Then the Corestriction Principle holds for the image of α_k with respect to Cores $k'/k, T$.
b) Let $\alpha_k : \mathrm{H}^p_{fppf}(k, T) \to \mathrm{H}^q_{fppf}(k, G)$ be a connecting map induced by an exact sequence involving k-groups. Assume that G is connected, reductive and T is a torus. Then the Corestriction Principle holds for the kernel of α_k with respect to Cores $k'/k, T$.

In the case of characteristic 0, Theorem 3 was known earlier (cf. [T2]). In next sections we indicate the main ingredients and results used in the proofs of our theorems.

3. *z*-extensions. As in the case of fields, for a ring A as above, and an exact sequence $1 \to Z \to H \to G \to 1$ of reductive A-group schemes, with Z an A-torus, we say that H is a *z-extension* of G if Z is an induced A-torus and the derived subgroup of H is simply connected (cf. [SGA 3], Exp. XXII, Sec. 4.3.3, [Ha] for the corresponding notions). If $x \in \mathrm{H}^1_{et}(A', G)$, we say that a z-extension $H \to G$ (over A) is *x-lifting* if $x \in \mathrm{Im}\,(\mathrm{H}^1_{et}(A', H_{A'}) \to \mathrm{H}^1_{et}(A', G_{A'}))$. We need the following assertion,

1) By convention, in the case of global function field k, we call the ring of regular functions of a smooth projective curve with function field k also the ring of integers of k.

whose proof is similar to the case of fields.

Proposition 1. *a*) ([Ha], Lemma 1.4.1) *With notation as above we have*

$$\mathrm{H}^1_{fppf}(A, T) = \mathrm{H}^1_{fppf}(A_1, \mathbf{G}^r_m).$$

b) *With notations as above, for any given reductive A-group scheme G, there exist z-extensions of G.*
c) *Given an exact sequence* $1 \to G_0 \to G_1 \to G_2 \to 1$ *of reductive A-group schemes, there exists a z-extension of this sequence, i.e., an exact sequence* $1 \to H_0 \to H_1 \to H_2 \to 1$ *of reductive A-group schemes and a commutative diagram*

$$\begin{array}{ccccccccc} 1 & \to & H_0 & \to & H_1 & \to & H_2 & \to & 1 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 1 & \to & G_0 & \to & G_1 & \to & G_2 & \to & 1 \end{array}$$

of reductive A-group schemes such that each A-group scheme H_i *is a z-extension of* G_i, $i = 0, 1, 2$.
d) *Let* A' *belong to* $\mathcal{C}_A$, *G a reductive A-group scheme. Then for any element* $x \in \mathrm{H}^1_{et}(A', G_{A'})$ *there exists a x-lifting z-extension H of G.*
e) *Let* A' *be as above and let* $\pi : G_1 \to G_2$ *be a morphism of reductive A-group schemes. Then for any given* $x \in \mathrm{H}^1_{et}(A', G_{1A'})$ *there exists a z-extension* $\pi' : H_1 \to H_2$ *of* $\pi : G_1 \to G_2$, *such that* H_1 *is x-lifting z-extension of* G_1.

Notice that *b*) above is an extension of a "cross diagram" lemma due to Ono [O] (cf. [Ha]), and *c*), *d*), *e*) extend some results due to Borovoi and Kottwitz (cf. [Bo1, Bo2] and references there).

4. Deligne hypercohomology and abelianized cohomology. In [De], Sec. 2.4, Deligne has associated to each pair $f : G_1 \to G_2$ of algebraic groups defined over a field k, where f is a k-morphism, a category $[G_1 \to G_2]$ of G_2-trivialized G_1-torsors, and certain hypercohomology sets, which can be done for sheafs of groups over any topos (loc.cit., p. 276). We denote them by $\mathbf{H}^i_r(G_1 \to G_2)$ for $i = -1, 0$, where r stands for étale or flat topology, which agrees with the notations of [Bo3] and [Br] (while in [De], the degree of the hypercohomology sets corresponding to $G_1 \to G_2$ is shifted). Then Borovoi in [Bo3] and Breen in [Br] gave a detailed exposition and extension of such hypercohomology and in [Bo3] (resp. [Br]), there was defined also the set $\mathbf{H}^1(G_1 \to G_2)$ (resp. $\mathbf{H}^1_r(G_1 \to G_2)$), where the setting in [Br] works over any topos T_r. In the case of a field of characteristic 0, the theory coincides with the one given by Borovoi ([Bo3]). As in [Bo3], by using [Br], we may also define the abelianization map $ab_G : \mathrm{H}^i_r(A, G) \to \mathbf{H}^i(\tilde{G} \to G)$, for a reductive A-group scheme, where $\tilde{G}$ is the simply connected semisimple A-group scheme, which is the universal covering of $G' = [G, G]$, the semisimple part of G, and $i = 0, 1$. In fact, it has been proved that if $\tilde{Z}$ (resp. Z) is the center of $\tilde{G}$ (resp. of G), then there are an equivalence of categories $[\tilde{Z} \to Z] \simeq [\tilde{G} \to G]$, and quasi-isomorphisms of complexes

$$(\tilde{Z} \to Z) \simeq (\tilde{T} \to T) \simeq (\tilde{G} \to G),$$

where $\tilde{T}$ (resp. T) is a maximal A-torus of $\tilde{G}$ (resp. G), with $f^{-1}(T) = \tilde{T}$. One defines $\mathrm{H}^i_{ab,r}(A, G) := \mathbf{H}^i_r(\tilde{G} \to G)$ and call it the *abelianized cohomology* of degree i of G (in the corresponding topos; here r stands for "ét" or "fppf" (if $\tilde{Z}$ is not smooth)).

5. Equivalent conditions for Corestriction principle. Let G be a reductive A-group scheme. Denote by G' the derived subgroup scheme of G, $\tilde{G}$ the simply connected covering of G', and $Ad(G)$ the adjoint group scheme of G (see [SGA 3], Exp. XXII, 4.3.3). Let $\tilde{F} :=$ Ker $(\tilde{G} \to Ad(G))$, $F :=$ Ker $(\tilde{G} \to G')$ and let $\tilde{Z}$, Z be as above. Since $\tilde{Z}$ and Z are commutative, the resulting cohomology sets $\mathbf{H}^i_r(\tilde{Z} \to Z)$ have natural structure of abelian groups. In the case of fields, it is known that there exists functorial corestriction homomorphisms for $\mathrm{H}^i_{ab,r}(A, G)$ (cf. [Pe, T2]). However, in the general case, it is not clear whether such functorial homomorphisms always exist. Thus we make the following assumption.

(Hyp_A) *For* $A' \in \mathcal{C}_A$, *for any G as above, with* $\tilde{Z}$ *smooth, there exist functorial corestriction homomorphisms* $Cores_{A'/A,ab} : \mathrm{H}^i_{ab,et}(A', G_{A'}) \to \mathrm{H}^i_{ab,et}(A, G), i = 0, 1$.

Let $\alpha : \mathrm{H}^p_{et}(A, G) \to \mathrm{H}^q_{et}(A, T)$ be a connecting map of cohomologies and assume that an extension A'/A, $A' \in \mathcal{C}_A$, is fixed. Under the assumption of (Hyp_A), we consider the following statements.

a) *The* (*Weak*) *Corestriction principle holds for the image of any connecting map* $\alpha : \mathrm{H}^p_{et}(A, G) \to \mathrm{H}^q_{et}(A, T)$ *for reductive A-group schemes G, T, with T an A-torus,* $0 \le p \le 1, p \le q \le 2$, *with respect to* $Cores_{A'/A,T}$.
b) *For any reductive A-group scheme, the* (*Weak*) *Corestriction principle holds for the image of functorial map* $ab_G : \mathrm{H}^p_{et}(A, G) \to \mathrm{H}^p_{ab,et}(A, G)$, $0 \le p \le 1$, *with respect to* $Cores_{A'/A,ab}$.
c) *The* (*Weak*) *Corestriction principle holds for*

the image of any connecting (coboundary) map $\mathrm{H}^p_{et}(A, G) \to \mathrm{H}^{p+1}_{et}(A, T), 0 \le p \le 1$, *with respect to* $Cores_{A'/A,T}$, *where* $1 \to T \to G_1 \to G \to 1$ *is any exact sequence of reductive A-group schemes, and T is a central smooth A-subgroup scheme.*

d) The same statement as in c), but G_1, G *are semisimple.*

e) The (Weak) Corestriction principle holds for the image of any connecting (coboundary) map $\mathrm{H}^p_{et}(A, Ad(G)) \to \mathrm{H}^{p+1}_{et}(A, F)$ *with respect to* $Cores_{A'/A,F}$, *where* $1 \to F \to G \to Ad(G) \to 1$ *is any exact sequence of A-group schemes, with F a central smooth A-subgroup of semisimple A-group G.*

f) The (Weak) Corestriction principle holds for the image of any connecting (coboundary) map $\mathrm{H}^p_{et}(A, Ad(G)) \to \mathrm{H}^{p+1}_{et}(A, F)$ *with respect to* $Cores_{A'/A,F}$, *where* $1 \to F \to G \to Ad(G) \to 1$ *is any exact sequence of A-group schemes, with F a central smooth A-subgroup of semisimple simply connected A-group G.*

Notice that we always have obvious implications $c) \Rightarrow d) \Rightarrow e) \Rightarrow f)$. For the statements above, denote by $x(p)$ (resp. $x(p,q)$) the corresponding statement evaluated at p (resp. at p, q). For example $a(0,1)$ means the statement $a)$ with $p = 0, q = 1$. We have the following theorem, the proof of which is similar to that of Theorem 2.10 in [T3].

Theorem 4. *a) Assuming* (Hyp_A), *there are the following equivalence relations*

$$a) \Leftrightarrow b), \quad c) \Leftrightarrow d), \quad e) \Leftrightarrow f).$$

b) The following relations between above statements for certain values of p,q hold. For low dimension we have

$$a(0,1) \Leftarrow a(0,0) \Leftrightarrow b(0) \Leftrightarrow c(0) \Leftrightarrow d(0) \Leftrightarrow e(0) \Leftrightarrow f(0).$$

For higher dimension we have

$$a(1,2) \Leftarrow a(1,1) \Leftrightarrow b(1),$$
$$c(1) \Leftrightarrow d(1) \Rightarrow e(1) \Leftrightarrow f(1),$$

and if A is a ring such that $\mathrm{H}^1_{et}(A', T_{A'}) = 0$ *for any induced A′-torus T,* $A' \in \mathcal{C}_A$, *then the following implications*

$$a(1,2) \Leftarrow a(1,1) \Leftrightarrow b(1) \Rightarrow c(1) \Leftrightarrow d(1) \Rightarrow e(1) \Leftrightarrow f(1)$$

hold true.

c) In general, without assuming (Hyp_A), *by ignoring conditions* $b(i)$, *all above implications hold true.*

6. Analogs of results of Kneser. The proof of Theorems 1, 2, and 3 makes use of results of [T2, T3] and, among other things, the following result, which is an analog of some results of Kneser [Kn] in the $\mathfrak{p}$-adic and number fields case.

Proposition 2. *a) Let G be a semisimple group over a local or global function field k,* $\pi : \tilde{G} \to G$ *the universal covering of G,* $F = \mathrm{Ker}\,(\pi)$. *Then the coboundary map* $\Delta_k : \mathrm{H}^1_{fppf}(k, G) \to \mathrm{H}^2_{fppf}(k, F)$ *is bijective.*

b) Let A be the ring of integers of a local non-archimedean field k, G a semisimple A-group, $\tilde{G}$ *the simply connected A-group scheme which is covering G, F the kernel of canonical morphism* $\tilde{G} \to G$. *Then the coboundary map* $\Delta : \mathrm{H}^1_{fppf}(A, G) \to \mathrm{H}^2_{fppf}(A, F)$ *is bijective.*

c) ([Do]) *Assume that A is a ring of integers of a global field k, G a semisimple A-group scheme,* $\tilde{G}$ *the simply connected A-group scheme which is covering G, F the kernel of canonical morphism* $\tilde{G} \to G$. *Then the coboundary map* $\Delta : \mathrm{H}^1_{fppf}(A, G) \to \mathrm{H}^2_{fppf}(A, F)$ *is surjective.*

d) With notation as in c), assume further that A is the ring of integers of global function field k. Then Δ *is bijective.*

In the case of any local (resp. global field), the bijectivity (resp. surjectivity) of Δ_k has been proved in [Do], which makes use of theory of bands (gerbes).

7. Serre - Grothendieck conjecture. Besides, we make also use of some results of Tits and Nisnevich related with Grothendieck - Serre conjecture. Let S be an integral, regular, Noetherian scheme with function field K, G a reductive group scheme over S. The Serre - Grothendieck conjecture (according to a version presented in [Ni]), stated that the sequence of (pointed) cohomology sets $1 \to \mathrm{H}^1_{Zar}(S, G) \to \mathrm{H}^1_{et}(S, G) \to \mathrm{H}^1(K, G_K)$ is exact. Equivalently, it says that

If S, G are as above, η *is the generic point of S and* $A = \mathcal{O}_x$ *is any local ring at* $x \in S \setminus \{\eta\}$, *then the natural map of cohomology* $\mathrm{H}^1_{et}(A, G) \to \mathrm{H}^1(K, G_K)$ *has trivial kernel.*

The results we need are due to Tits (unpublished, but see [Ni], Theorem 4.1) and to Nisnevich [Ni] (Theorem 4.5), which confirm Serre - Grothendieck conjecture for Dedekind and local henselian rings.

8. Applications. With k, A, G, T as in Theorem 2, assume $\pi : G \to T$ is a morphism of A-group schemes. Denote by G' the derived subgroup of G, $k_\infty = \prod_{v \in \infty} k_v$, $\mathbf{A}$ and $\mathbf{A}(\infty)$ the adèle ring of k and the subring of integral adèles of $\mathbf{A}$, respectively. Let $Cl_A(G) = G(\mathbf{A}(\infty)) \setminus G(\mathbf{A})/G(k)$ be the set of double classes of the adèle group $G(\mathbf{A})$, which is an important arithmetic invariant of G (see, e.g., [De, Gi, Ha, KS, O]). As an application of above results and, among other things, some results due to Nisnevich, Kato and Saito [KS], we have

Theorem 5. *The Corestriction principle holds for the image of the induced map of class sets* $\pi_A : Cl_A(G) \to Cl_A(T)$ *with respect to norm* $N : Cl_{A'}(T) \to Cl_A(T)$. *If, moreover,* $G'(k_\infty)$ *is non-compact, then* $Cl_A(G)$ *has a natural finite abelian group structure and there is a norm homomorphism*

$$N_{A'/A} : Cl_{A'}(G_{A'}) \to Cl_A(G)$$

for any $A' \in \mathcal{C}_A$, *which is functorial in* A'.

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