Existence result for a doubly degenerate quasilinear stochastic parabolic equation

By Mamadou SANGO

Department of Mathematics and Applied Mathematics, University of Pretoria
Mamelodi Campus, Private Bag X1311, Silverton 0127, Tswane, South Africa

(Communicated by Heisuke HIRONAKA, M. J. A., May 12, 2005)

Abstract: Using the splitting-up method, we establish a new existence result for an initial boundary value problem for the doubly degenerate stochastic quasilinear parabolic equation

$$d\left(|y|^{\alpha-2}y\right)-\left[\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\left|\frac{\partial y}{\partial x}\right|^{p-2}\frac{\partial y}{\partial x_i}\right)-g(t,y)\right]dt=\sum_{l=0}^{d}h_l(t,y)dW_t^l,$$

where W_t^l are one-dimensional Wiener process defined on a complete probability space, p, α and the functions g and h_l satisfy appropriate restrictions.

Key words: Doubly degenerate; stochastic; parabolic equations; splitting-up method; compactness.

1. Introduction. Let D be a bounded domain in the Euclidean space $\mathbf{R}^n$ $(n \geq 2)$ with a sufficiently smooth boundary ∂D. For $0 < T < \infty$, we denote by Q_T the cylinder $D \times (0,T)$. Let $\left(\boldsymbol{\Omega}, \mathbf{F}, \{\mathbf{F}_t\}_{0\leq t\leq T}, \mathbf{P}\right)$ be a complete filtered probability space on which are defined the one-dimensional Wiener processes W_t^l such that $(\{\mathbf{F}_t\}, 0 \leq t \leq T)$ is a natural filtration of $W_t^l(t)$, augmented by all $\mathbf{P}$-negligeable sets, we denote by ω the sample points from $\boldsymbol{\Omega}$. In $Q_T \times \boldsymbol{\Omega}$, we investigate the initial boundary value problem for the doubly degenerate quasilinear stochastic parabolic equation

$$(1)\qquad d\left(|y|^{\alpha-2}y\right)-\left[\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\left|\frac{\partial y}{\partial x}\right|^{p-2}\frac{\partial y}{\partial x_i}\right)-g(t,y)\right]dt=\sum_{l=0}^{d}h_l(t,y)dW_t^l, \quad \text{in } Q_T\times\boldsymbol{\Omega},$$

$$(2)\qquad y(x,t,\omega)=0 \quad \text{on} \quad \partial D\times(0,T)\times\boldsymbol{\Omega},$$

$$(3)\qquad y(x,0,\omega)=y_0(x,\omega) \quad \text{in} \quad D\times\boldsymbol{\Omega}.$$

We assume that there exists some non negative constants c_1, c_2, c_3^l, c_4^l such that the functions g and h_l satisfy

$$(4)\qquad c_1|y|^{2\mu-2}y \leq g(t,y) \leq c_2|y|^{2\mu-2}y,$$

$$(5)\qquad c_3^l|y|^{\sigma_l-1}y \leq h_l(t,y) \leq c_4^l|y|^{\sigma_l-1}y,$$

and α, μ, σ_l $(l = 1,\dots,d)$ and p are some nonnegative numbers satisfying the restrictions: $1 < \alpha \leq 2$, $p \geq 2$,

$$(6)\qquad \sigma_l \leq \min\{2\mu/\alpha', \alpha-1\},\ \forall l;\ p \geq 2\mu \geq 1.$$

Here and in the sequel r' will stand for the Hölder conjugate of a number $r > 1$, i.e., $r' = r/(r-1)$, $\partial u/\partial x$ denotes the gradient vector of u. From now on, we supress the dependence on t when writing g and h_l. Denote

$$A(y)=-\sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\left|\frac{\partial y}{\partial x}\right|^{p-2}\frac{\partial y}{\partial x_i}\right).$$

The aim of the present work is to establish an existence result for the problem (1)–(3). The equation (1) is degenerate when $y = 0$ or when the gradient $\partial y/\partial x$ vanishes. In the deterministic case (i.e., $c_4^l = 0$), equations of this type are known to be of great importance in the applied sciences. Indeed they model processes ranging from the theory of non newtonian fluids to population dynamics; detailed informations can be found in the survey [4] by Kalashnikov. Initial boundary value problems of equation (1) are naturally expected to describe some stochastic counterparts of the deterministic known models.

2000 Mathematics Subject Classification. 60H15, 35R60.

In the deterministic case existence results for degenerate quasilinear parabolic equations were obtained by many authors (see e.g., [1, 3, 9, 12], just to mention a few). For existence results of monotone stochastic nonlinear parabolic equations (when $\alpha = 2$) we refer for instance to [2, 5, 7, 8].

We use the splitting-up method which seems to be the most effective tool for establishing the existence of a solution to problem (1)–(3). The traditional methods of semi-discretization as in [9] or Galerkin's method as in [8], or in [5] appear to be ineffective; the nature of the nonlinearity in equation (1) coupled with the presence of the stochastic nonlinear term seems to be the main stumbling block. The simpler case when $p = 2$ was considered by the author in [10]. The full-length version of this note will appear elsewhere.

2. Notations, formulation of main and auxiliary results. We consider the well-known function spaces $L_q(D)$, $W_q^1(D)$, $\overset{o}{W}{}_q^1(D)$ with $q \geq 1$. By $W_{q'}^{-1}(D)$, we denote the dual of $\overset{o}{W}{}_q^1(D)$ for $q > 1$. Let X be a Banach space. For $r, q \geq 1$, we denote by $\mathcal{L}_{r,q}(0, T, \mathbf{\Omega}, X)$ the space of functions $u = u(x, t, \omega)$ with values in X defined on $[0, T] \times \mathbf{\Omega}$ and such that

1) u is measurable with respect to (t, ω) and for each t, u is $\mathcal{F}_t$-measurable.

2) $u \in X$ for almost all (t, ω) and

$$\|u\|_{\mathcal{L}_{r,q}(0,T,\mathbf{\Omega},X)} = \left(E\left(\int_0^T \|u\|_X^q \, dt \right)^{r/q} \right)^{1/r} < \infty,$$

where E denotes the mathematical expectation.

The space $\mathcal{L}_{r,q}(0, T, \mathbf{\Omega}, X)$ so defined is a Banach space. If $r = q$, we shall simply write $\mathcal{L}_r(0, T, \mathbf{\Omega}, X)$ for $\mathcal{L}_{r,r}(0, T, \mathbf{\Omega}, X)$. When $q = \infty$, the norm in $\mathcal{L}_{r,\infty}(0, T, \mathbf{\Omega}, X)$ is given by

$$\|u\|_{\mathcal{L}_{r,\infty}(0,T,\mathbf{\Omega},X)} = \left(E \sup_{0 \leq t \leq T} \|u\|_X^r \right)^{1/r}.$$

The space of functions u with values in X defined on $[0, T] \times \mathbf{\Omega}$ such that

$$\sup_{0 \leq t \leq T} E \|u(t)\|_X^q < \infty$$

is denoted by $\mathcal{L}_{\infty,q}(0, T, \mathbf{\Omega}, X)$. For $q \geq 1$, we also consider the space $L_q(0, T, X)$ of functions $\varphi = \varphi(x, t)$ with values in X defined on $[0, T]$ and such that

$$\|u\|_{L_q(0,T,X)} = \left(\int_0^T \|u\|_X^q \, dt \right)^{1/q} < \infty.$$

Definition 1. By a solution of problem (1)–(3) we shall mean a function $y(t, \omega)$ such that

1.
$$y \in \mathcal{L}_p\Big(0, T, \mathbf{\Omega}, \overset{o}{W}{}_p^1(D)\Big) \cap \mathcal{L}_{\alpha,\infty}(0, T, \mathbf{\Omega}, L_\alpha(D)) \cap \mathcal{L}_{2\mu}(0, T, \mathbf{\Omega}, L_{2\mu}(D));$$

2. for almost all $(t, \omega) \in [0, T] \times \mathbf{\Omega}$, the relation

$$|y(t,\omega)|^{\alpha-2} y(t,\omega) - |y_0(t,\omega)|^{\alpha-2} y_0 + \int_0^t A(y(\tau))\, d\tau + \int_0^t g(y(\tau))\, d\tau = \sum_{l=0}^d \int_0^t h_l(y(\tau))\, dW_\tau^l, \tag{7}$$

holds as an equality of elements of $W_p^{-1}(D)$.

In this work we prove

Theorem 2. *Let the conditions* (4), (5) *and* (6) *be satisfied. Assume that*

$$P\Big\{\omega : y_0 \in \overset{o}{W}{}_p^1(D) \cap L_\alpha(D) \cap L_{2\mu}(D)\Big\} = 1.$$

Then there exists a function

$$y \in \mathcal{L}_p\Big(0, T, \mathbf{\Omega}, \overset{o}{W}{}_p^1(D)\Big) \cap \mathcal{L}_{\alpha,\infty}(0, T, \mathbf{\Omega}, L_\alpha(D)) \cap \mathcal{L}_{2\mu}(0, T, \mathbf{\Omega}, L_{2\mu}(D)),$$

solution of the problem (1)–(3) *for almost every* $\omega \in \mathbf{\Omega}$.

Remark 3. Arguing as in [5, 8], it is possible to show that $|y|^{\alpha-2} y$ has a continuous modification in $t \in [0, T]$. Therefore y also has a continuous modification in t. Consequently by identifying y with its continuous modification, we see that the initial condition (3) is meaningful. In the sequel we shall assume such an identification.

The proof of the theorem will involve the following two compactness results. In analogy with [6](Chap. 1, Lemma 1.3) we have

Lemma 4. *Let* $(g_\kappa)_{\kappa=1,2\ldots}$ *and* g *be some functions in* $\mathcal{L}_q(0, T, S, L_q(D))$ *with* $q \in (1, \infty)$ *such that*

$$\|g_\kappa\|_{\mathcal{L}_q(0,T,S,L_q(D))} \leq C, \forall \kappa$$

and as $\kappa \to \infty$

$$g_\kappa \to g \text{ for almost all } (x, t, \omega) \in Q_T \times \mathbf{\Omega}.$$

Then g_κ *weakly converges to* g *in* $\mathcal{L}_q(0, T, \mathbf{\Omega}, L_q(D))$.

The next result is from [11] (Section 5, Theorem 5 and Remark 8.2). It refines an earlier result due to Dubinskii [3].

Lemma 5. *Let B and B_1 be some Banach spaces such that B is continuously embedded into B_1 and let Y be a subset of B such that $\lambda Y \subset Y$ for all $\lambda \in \mathbf{R}$. Assume that Y is endowed with the semi-norm $Y \ni v \to M(v) \in \mathbf{R}_+$ such that*

$$M(\lambda v) = |\lambda| M(v),$$

and the set $\{v : v \in Y,\ M(v) \leq 1\}$ is relatively compact in B.

Let q, $q_1 \in (1, \infty)$. We define $L_q(0, T, Y)$ to be the set of functions measurable with respect to t, and with values in Y such that

$$\|u\|_{L_q(0,T,Y)} = \left(\int_0^T M[v(t)]^q \, dt \right)^{1/q} < C_1.$$

Let V be a set bounded in $L_q(0, T, Y)$ such that

$$\lim_{\theta \to 0} \int_0^{T-\theta} \|v(t+\theta) - v(t)\|_{B_1}^{q_1} \, dt = 0,$$

uniformly for all $v \in V$. Then V is relatively compact in $L_q(0, T, B)$.

3. The splitting-up algorithm and a priori estimates. As from now we agree to denote all non essential positive constants by C. Let $k = T/(N+1)$, where $N = 0, 1, 2, \ldots$. We split the interval $[0, T]$ into subintervals $[rk, (r+1)k[, r = 0, \ldots, N$ and consider the number $\rho \in (0, 1)$.

On the probability space $\left(\mathbf{\Omega}, \mathbf{F}, \{\mathbf{F}_t\}_{0 \leq t \leq T}, \mathbf{P}\right)$ we define the stochastic processes y_k and $\tilde{y}_k$ such that for $t \in (rk, (r+1)k)$

$$\frac{\partial\left(|y_k|^{\alpha-2} y_k\right)}{\partial t} + A(y_k) + (1-\rho) g(y_k) = 0, \tag{8}$$

$$y_k(rk) = y_k^r, \tag{9}$$

$$d\left(|\tilde{y}_k|^{\alpha-2} \tilde{y}_k\right) + \rho g(\tilde{y}_k) \, dt = \sum_{l=0}^{d} h_l(y_k) \, dW_t^l, \tag{10}$$

$$\tilde{y}_k(rk) = y_k((r+1)k - 0). \tag{11}$$

Next we set

$$y_k^{r+1} = \tilde{y}_k((r+1)k - 0), \tag{12}$$

and we start with

$$y_k^0 = \tilde{y}_k^0 = y_0; \tag{13}$$

the processes y_k and $\tilde{y}_k$ are subject to the homogenous Dirichlet boundary condition (2) on $\partial D \times (0, T) \times \mathbf{\Omega}$. The first problem is deterministic and is known to be solvable under the boundary conditions (2) (see e.g., [12]). By the substitution

$$w(t) = |\tilde{y}_k(t)|^{\alpha-2} \tilde{y}_k(t),$$

equation (10) reduces to

$$dw + \rho g(|w|^{\gamma} w) \, dt = \sum_{l=0}^{d} h_l(y_k) \, dW_t^l, \quad t \in (rk, (r+1)k), \tag{14}$$

where $\gamma = (2-\alpha)/(\alpha-1)$. In order to show that this equation has a unique solution we use the vanishing viscosity method coupled with the existence result of [5]. The problem (8)–(9) is not uniquely solvable in general. However the extraction of a suitable subsequence through a diagonalization process will do the job. In the sequel we shall denote any such subsequence also by the symbol y_k.

We have

Lemma 6. *If*

$$P\left\{\omega : \tilde{y}_k^0,\ y_k^0 \in \overset{o}{W}{}_p^1(D) \cap L_\alpha(D) \cap L_{2\mu}(D)\right\} = 1,$$

and the conditions of Theorem 2 are met, then the processes y_k and $\tilde{y}_k$ satisfy the following statements. The quantities

$$\sup_{0 \leq t \leq T} E\|y_k(t)\|_{L_\alpha(D)}^\alpha + E\int_0^T \|y_k(s)\|_{\overset{o}{W}{}_p^1(D)}^p \, ds + E\int_0^T \|y_k(s)\|_{L_{2\mu}(D)}^{2\mu} \, ds, \tag{15}$$

$$\sup_{0 \leq t \leq T} E\|\tilde{y}_k(t)\|_{L_\alpha(D)}^\alpha + E\int_0^T \|\tilde{y}_k(s)\|_{L_{2\mu}(D)}^{2\mu} \, ds, \tag{16}$$

are finite.

$$E\int_0^{T-\theta} \left\|\Delta\left[|y_k|^{\alpha-2} y_k(t)\right]\right\|_{W_{p'}^{-1}(D)}^{p'} dt = O(\theta), \tag{17}$$

$$E\int_0^{T-\theta} \left\|\Delta\left[|\tilde{y}_k|^{\alpha-2} \tilde{y}_k(t)\right]\right\|_{W_{p'}^{-1}(D)}^{p'} dt = O(\theta), \tag{18}$$

$$E\left\||y_k(t)|^{\alpha-2} y_k(t) - |\tilde{y}_k(t)|^{\alpha-2} \tilde{y}_k(t)\right\|_{W_{p'}^{-1}(D)}^{p'} \leq Ck, \tag{19}$$

for all $t \in [0, T]$, where $O(\theta) \to 0$ as $\theta \to 0$ and

$$\Delta\left[|z_k|^{\alpha-2} z_k(t)\right] =: |z_k|^{\alpha-2} z_k(t+\theta) - |z_k|^{\alpha-2} \tilde{y}_k(t).$$

Furthermore for every $q \geq \alpha$, we have

$$E \sup_{0 \leq t \leq T} \|y_k(t)\|^{2q}_{L_{2q}(\Omega)} + E \sup_{0 \leq t \leq T} \|\tilde{y}_k(t)\|^{2q}_{L_{2q}(\Omega)} \leq C, \ \forall t \in [0,T]. \tag{20}$$

4. Sketch of the proof of Theorem 2. By Lemma 6, we have the following convergences as $k \to 0$ (after extracting a suitable subsequence from y_k and $\tilde{y}_k$ that we denote again by the same symbols)

$$y_k \rightharpoonup y \ \text{ weakly}^* \text{ in } \ \mathcal{L}_{\infty,\alpha}\left(0,T,\mathbf{\Omega},L_\alpha(D)\right), \tag{21}$$

$$y_k(T) \rightharpoonup \bar{y} \ \text{ weakly}^* \text{ in } \ L_\alpha(D), \tag{22}$$

for almost all ω,

$$y_k \rightharpoonup y \ \text{ weakly in } \ \mathcal{L}_{2\mu}\left(0,T,\mathbf{\Omega},L_{2\mu}(D)\right), \tag{23}$$

$$y_k \rightharpoonup y \ \text{ weakly in } \ \mathcal{L}_p\Big(0,T,\mathbf{\Omega},\overset{o}{W}{}^1_p(D)\Big), \tag{24}$$

$$\tilde{y}_k \rightharpoonup \tilde{y} \ \text{ weakly}^* \text{ in } \ \mathcal{L}_{\infty,\alpha}\left(0,T,\mathbf{\Omega},L_\alpha(D)\right), \tag{25}$$

$$\tilde{y}_k \rightharpoonup \tilde{y} \ \text{ weakly in } \ \mathcal{L}_{2\mu}\left(0,T,\mathbf{\Omega},L_{2\mu}(D)\right), \tag{26}$$

$$y_k \rightharpoonup y \ \text{ weakly}^* \text{ in } \ \mathcal{L}_{\lambda,\infty}\left(0,T,\mathbf{\Omega},L_\lambda(D)\right), \tag{27}$$

$$\tilde{y}_k \rightharpoonup \tilde{y} \ \text{ weakly}^* \text{ in } \ \mathcal{L}_{\lambda,\infty}\left(0,T,\mathbf{\Omega},L_\lambda(D)\right), \tag{28}$$

where $\lambda \geq 2\alpha$. Thus

$$y \in \mathcal{L}_p\Big(0,T,\mathbf{\Omega},\overset{o}{W}{}^1_p(D)\Big) \cap \mathcal{L}_{\infty,\alpha}\left(0,T,\mathbf{\Omega},L_\alpha(D)\right) \cap \mathcal{L}_{2\mu}\left(0,T,\mathbf{\Omega},L_{2\mu}(D)\right),$$

and

$$\tilde{y} \in \mathcal{L}_{2\mu}\left(0,T,\mathbf{\Omega},L_{2\mu}(D)\right) \cap \mathcal{L}_{\infty,\alpha}\left(0,T,\mathbf{\Omega},L_\alpha(D)\right).$$

Furthermore

$$\sup_{0\leq t\leq T} E\,\|y(t)\|^\alpha_{L_\alpha(D)} + E\int_0^T \|y(s)\|^p_{\overset{o}{W}{}^1_p(D)}\,ds + E\int_0^T \|y(s)\|^{2\mu}_{L_{2\mu}(D)}\,ds,$$

$$\sup_{0\leq t\leq T} E\,\|\tilde{y}(t)\|^\alpha_{L_\alpha(D)} + E\int_0^T \|\tilde{y}\|^{2\mu}_{L_{2\mu}(D)}\,ds$$

are finite and

$$E\int_0^{T-\theta} \left\|\Delta\left[|y|^{\alpha-2}\,y(t)\right]\right\|^{p'}_{W^{-1}_{p'}(D)} dt = O(\theta),$$

$$E\int_0^{T-\theta} \left\|\Delta\left[|\tilde{y}|^{\alpha-2}\,\tilde{y}(t)\right]\right\|^{p'}_{W^{-1}_{p'}(D)} dt = O(\theta),$$

$$E\left\|\left|y(t)\right|^{\alpha-2}y(t) - \left|\tilde{y}(t)\right|^{\alpha-2}\tilde{y}(t)\right\|^{p'}_{W^{-1}_{p'}(D)} \leq Ck,$$

$$E\sup_{0\leq t\leq T}\|y(t)\|^\lambda_{L_\lambda(\Omega)} + E\sup_{0\leq t\leq T}\|\tilde{y}(t)\|^\lambda_{L_\lambda(\Omega)} \leq C.$$

We note that by (6) $(2\mu)' \geq p'$. Thus $g\left(y_k\right)$ is bounded in $\mathcal{L}_{p'}\left(0,T,\mathbf{\Omega},L_{p'}(D)\right)$; also $\sigma_i \leq \alpha-1$, hence by Lemma 6 $h_l\left(y_k\right)$ is bounded in $L_\alpha(D)$ for almost all t, ω and $l = 1,\ldots,d$. From the above convergences it therefore follows that there exist the functions v, $\tilde{v}$, f, φ, $\tilde{\varphi}$, ψ_l $(l=1,\ldots,d)$ such that

$$\left|y_k\right|^{\alpha-2}y_k \to v \text{ weakly}^* \text{ in } \mathcal{L}_{\alpha',\infty}\left(0,T,\mathbf{\Omega},L_{\alpha'}(D)\right), \tag{29}$$

$$A\left(t,y_k(t)\right) \rightharpoonup f \text{ weakly in } \mathcal{L}_{p'}\big(0,T,\mathbf{\Omega},W^{-1}_{p'}(D)\big), \tag{30}$$

$$\left|\tilde{y}_k\right|^{\alpha-2}\tilde{y}_k \to \tilde{v} \text{ weakly}^* \text{ in } \mathcal{L}_{\alpha',\infty}\left(0,T,\mathbf{\Omega},L_{\alpha'}(D)\right), \tag{31}$$

$$g\left(\tilde{y}_k\right) \rightharpoonup \tilde{\varphi} \text{ weakly in } \mathcal{L}_{p'}\left(0,T,\mathbf{\Omega},L_{p'}(D)\right), \tag{32}$$

$$g\left(y_k\right) \rightharpoonup \varphi \text{ weakly in } \mathcal{L}_{p'}\left(0,T,\mathbf{\Omega},L_{p'}(D)\right), \tag{33}$$

$$h_l\left(y_k\right) \rightharpoonup \psi_l \text{ weakly}^* \text{ in } \mathcal{L}_{\alpha',\infty}\left(0,T,\mathbf{\Omega},L_{\alpha'}(D)\right). \tag{34}$$

Further since any strongly continuous linear operator is weakly continuous, it follows from (34) that

$$\int_D \sum_{l=0}^d h_l\left(y_k\right) dW^l_t \rightharpoonup \int_D \sum_{l=0}^d \psi_l dW^l_t \quad \text{weakly in } \ L_{\alpha'}(D), \tag{35}$$

for almost all ω and t.

Next let

$$Y = \left\{v : |v|^{\alpha'-2}\,v \in \overset{o}{W}{}^1_p(D)\right\},$$

$$M(v) = \left[\int_D \left|\frac{\partial}{\partial x}\left(|v|^{\alpha'-2}\,v\right)\right|^p dx\right]^{1/\left[(\alpha'-1)p\right]},$$

and set $q = (\alpha'-1)\,p$. If $q > 1$, i.e., $\alpha < p+1$, it follows that the set $\{v : v \in Y,\ M(v) \leq 1\}$ is relatively compact in $L_q(D)$, since $\overset{o}{W}{}^1_q(D)$ is compactly embedded into $L_q(D)$. Since $p \geq \alpha$, a simple verification shows that $q \geq p'$. Thus $L_q(\Omega) \subset L_{p'}(\Omega) \subset W^{-1}_{p'}(\Omega)$. Therefore taking $B = L_q(D)$ and $B_1 = W^{-1}_{p'}(D)$, we get that B is continuously embedded into B_1. Setting $v_k = \left|y_k\right|^{\alpha-2}y_k$ we see from Lemma 6 that v_k lies in the set V defined in Lemma 5. Thus by Lemma 5 it follows that

$$v_k \to v \text{ strongly in } L_q(0,T,L_q(D)), \quad \text{for almost all } \omega. \tag{36}$$

Thus

$$v_k(.,\omega) \to v(.,\omega) \text{ for almost all } \omega, \text{ as } k \to \infty. \tag{37}$$

Let δ be such that $\delta > \max\{p, 2\alpha\}$. Then $\delta/(\alpha-1) > 2\alpha$ since $(\alpha-1) \in (0,1]$. Thus (20) implies

$$\sup_{0\le t\le T} E\,\|v_k(t)\|^{\delta/(\alpha-1)}_{L_{\delta/(\alpha-1)}\left(0,T,L_{\delta/(\alpha-1)}(D)\right)} < C. \tag{38}$$

Since $q = p/(\alpha-1) < \delta/(\alpha-1)$ it follows that $\|v_k(\omega)\|^q_{L_q(0,T,L_q(D))}$ is equintegrable with respect to ω. Then combining (37) and (38) and using Vitali's convergence theorem we have

$$v_k \to v \text{ strongly in } \mathcal{L}_q\left(0,T,\mathbf{\Omega},L_q(D)\right).$$

Consequently we get

$$v_k \to v \text{ for almost all } (x,t,\omega) \in Q_T \times \mathbf{\Omega}. \tag{39}$$

Now by (21), (39) and Lemma 4 we get that

$$v = |y|^{\alpha-2}\, y \text{ for almost all } (x,t,\omega) \in Q_T \times \mathbf{\Omega}. \tag{40}$$

Analogously we obtain that for almost all $(x,t,\omega) \in Q_T \times \mathbf{\Omega}$

$$\tilde{v} = |\tilde{y}|^{\alpha-2}\,\tilde{y}, \quad \varphi = g(y), \qquad \tilde{\varphi} = g(\tilde{y}), \quad \psi_l = h_l(y). \tag{41}$$

From (8)–(13) we see that y_k and $\tilde{y}_k$ satisfy the equations

$$|y_k(t)|^{\alpha-2}\, y_k(t) + \int_0^t A(y_k)\, ds + (1-\rho)\int_0^t g(y_k)\, ds + \rho\int_0^{k[t/k]} g(\tilde{y}_k)\, ds = \int_0^{k[t/k]} h(y_k)\, dW + |y_0|^{\alpha-2}\, y_0, \tag{42}$$

and

$$|\tilde{y}_k(t)|^{\alpha-2}\,\tilde{y}_k(t) + \rho\int_0^t g(\tilde{y}_k)\, ds + (1-\rho)\int_0^{k[t/k]+k} g(y_k)\, ds + \int_0^{k[t/k]+k} A(y_k)\, ds = \int_0^t h(y_k)\, dW_s + |y_0|^{\alpha-2}\, y_0, \tag{43}$$

where $h(y)dW_t =: \sum_{l=0}^d h_l(y)dW_t^l$. By the estimate (19), we have that

$$\mathbf{P}\left\{\omega : |y|^{\alpha-2}\, y = |\tilde{y}|^{\alpha-2}\,\tilde{y} \text{ for almost all } x,t\right\} = 1.$$

Hence by the monotonicity of the function $\zeta \to |\zeta|^{\alpha-2}\,\zeta$ we get

$$\mathbf{P}\Big\{\omega : y = \tilde{y} \text{ for almost all } x,t\Big\} = 1. \tag{44}$$

Arguing as in [8] (Theorem 3, Chap. 3, pp. 113–115) and taking in account the relations (21)–(35), (40), (41) and (44), we obtain by passage to the limit in (42) and (43) as $k \to 0$ that $\bar{y} = y(T)$, and for almost all $(x,t,\omega) \in Q_T \times \mathbf{\Omega}$

$$|y(t)|^{\alpha-2}\, y(t) + \int_0^t f\, ds + \int_0^t |y|^{2\mu-2}\, y\, ds = \int_0^t h(y_k)\, dW + |y_0|^{\alpha-2}\, y_0. \tag{45}$$

Finally we show that

$$f(x,t,\omega) = A(y), \text{ for almost all } (x,t,\omega) \in Q_T \times \mathbf{\Omega}.$$

Acknowledgements. The work was accomplished when the authour was visiting Tohoku University (Sendai, Japan) and IHES (Bures-sur-Yvette, France). The author expresses its deepest gratitude to these institutions for their hospitality and to Japan Society for Promotion of Science, Ernest Oppenheimer Memorial Trust, Fondation Schlumberger and University of Pretoria for their financial support.

References

[1] A. Bamberger, Étude d'une équation doublement non linéaire, J. Functional Analysis **24** (1977), no. 2, 148–155.

[2] A. Bensoussan, Some existence results for stochastic partial differential equations, in *Stochastic partial differential equations and applications* (*Trento, 1990*), 37–53, Pritman Res. Notes Math. Ser., 268, Longman Sci. Tech., Harlow, 1992.

[3] Ju. A. Dubinskiĭ, Weak convergence for nonlinear elliptic and parabolic equations, Mat. Sb. (N.S.) **67 (109)** (1965), 609–642.

[4] A. S. Kalashnikov, Some problems of the qualitative theory of second-order nonlinear degenerate parabolic equations, Uspekhi Mat. Nauk **42** (1987), no. 2, 135–176, 287.

[5] N. V. Krylov and B. L. Rozovskiĭ, Stochastic evolution equations, in *Current problems in mathematics,* XIV. (Russian) 256, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Informatsii, Moscow, 1979, pp. 71–147.

[6] J.-L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Gauthier-Villars, Paris, 1969.

[7] N. Nagase, Remarks on nonlinear stochastic partial differential equations: an application of the splitting-up method, SIAM J. Control Optim. **33** (1995), no. 6, 1716–1730.

[8] E. Pardoux, Equations aux derivées partielles stochastiques non linéaires monotones, Thèse, (1975). (Université Paris XI).

[9] P. A. Raviart, Sur la résolution de certaines équations paraboliques non linéaires, J. Functional Analysis **5** (1970), 299–328.

[10] M. Sango, On a degenerate quasilinear stochastic parabolic equation, C. R. Math. Acad. Sci. Soc. R. Can. **25** (2003), no. 3, 65–70.

[11] J. Simon, Compact sets in the space $L^p(0,T;B)$, Ann. Mat. Pura Appl. (4) **146** (1987), 65–96.

[12] M. Tsutsumi, On solutions of some doubly nonlinear degenerate parabolic equations with absorption, J. Math. Anal. Appl. **132** (1988), no. 1, 187–212.