L_p-L_q maximal regularity and viscous incompressible flows with free surface

By Yoshihiro SHIBATA*) and Senjo SHIMIZU**)

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Abstract: We prove the L_p-L_q maximal regularity of solutions to the Neumann problem for the Stokes equations with non-homogeneous boundary condition and divergence condition in a bounded domain. And as an application, we consider a free boundary problem for the Navier-Stokes equation. We prove a locally in time unique existence of solutions to this problem for any initial data and a globally in time unique existence of solutions to this problem for some small initial data.

Key words: Stokes equations; Neumann boundary condition; maximal regularity; Navier-Stokes equations; free boundary problem.

We consider a certain time dependent problem with free surface for the Navier-Stokes equations which describes the motion of an isolated finite volume of viscous incompressible fluid without taking surface tension into account. The region $\Omega_t \subset \mathbf{R}^n$, $n \geqq 2$, occupied by the fluid is given only on the initial time $t = 0$, while for $t > 0$ it is to be determined. The velocity vector field $v(x,t) = (v_1, \dots, v_n)^*$ and the pressure $\theta(x,t)$ for $x \in \Omega_t$ satisfy the Navier-Stokes equations (cf. [4]):

$$
(1) \qquad \begin{aligned}
&v_t + (v \cdot \nabla)v - \operatorname{Div} S(v,\theta) = f(x,t) && \text{in } \Omega_t,\ t > 0\\
&\operatorname{div} v = 0 && \text{in } \Omega_t,\ t > 0\\
&S(v,\theta)\nu_t + \theta_0(x,t)\nu_t = 0 && \text{in } \Gamma_t,\ t > 0\\
&v|_{t=0} = v_0 && \text{on } \Omega.
\end{aligned}
$$

Here, M^* denotes the transpose of M, Γ_t denotes the boundary of Ω_t and $\nu_t(x)$ is the unit outer normal to Γ_t at the point $x \in \Gamma_t$, and $\nabla = (\partial_1, \dots, \partial_n)$ with $\partial_i = \partial/\partial x_i$. $S(v,\theta)$ is the stress tensor defined by the formula:

$$S(v,\theta) = D(v) - \theta I,$$

where $D(v)$ is the deformation tensor of the velocities with elements $D_{ij}(v) = \partial_i v_j + \partial_j v_i$ and I is the $n \times n$ identity matrix. Writing $S = (S_{ij})$, we have set

$$\operatorname{Div} S = \left(\sum_{j=1}^n \partial_j S_{1j}, \dots, \sum_{j=1}^n \partial_j S_{nj} \right)^*.$$

The external force $f(x,t)$ and the pressure $\theta_0(x,t)$ are functions defined on the whole space. In what follows, we shall always assume that $\theta_0(x,t) = 0$, since we can arrive at this case by replacing $\theta(x,t)$ by $\theta + \theta_0$.

Aside from the dynamical boundary condition, a further kinematic condition for Γ_t is satisfied, from which it follows that Γ_t consists of points $x = x(\xi,t)$, $\xi \in \Gamma_0$, where $x(\xi,t)$ is the solution of the Cauchy problem:

$$(2) \qquad \frac{dx}{dt} = v(x,t), \quad x|_{t=0} = \xi.$$

This expresses the fact that the free surface Γ_t consists for all $t > 0$ of the same fluid particles, which do not leave it nor plunge into Ω_t. It is clear that $\Omega_t = \{x = x(\xi,t) \mid \xi \in \Omega_0\}$. We denote Ω_0 by Ω.

The problem (1) can therefore be written as an initial boundary value problem in the given region Ω_0 if we go over from the Euler coordinates $x \in \Omega_t$ to the Lagrange coordinates $\xi \in \Omega$ connected with x by (2). If a velocity vector field $u(\xi,t) = (u_1, \dots, u_n)^*$ is known as a function of the Lagrange coordinates ξ, then this connection can be written in the form:

$$x = \xi + \int_0^t u(\xi,\tau)\, d\tau \equiv X_u(\xi,t).$$

Passing to the Lagrange coordinate in (1) and setting $\theta(X_u(\xi,t),t) = \pi(\xi,t)$, we obtain

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*) Department of Mathematical Sciences, School of Science and Engineering, Waseda University, Ohkubo 3-4-1, Shinjuku-ku, Tokyo 169-8555.

**) Faculty of Engineering, Shizuoka University, Hamamatsu, Shizuoka 432-8561.

$$(3)\quad \begin{aligned} & u_t - \mathrm{Div}[S(u,\pi)+U(u,\pi)] = f(X_u(\xi,t),t) && \text{in } \Omega\times(0,T_0)\\ & \mathrm{div}\, u + E(u) = \mathrm{div}[u+\tilde{E}(u)] = 0 && \text{in } \Omega\times(0,T_0)\\ & [S(u,\pi)+U(u,\pi)]\nu = 0 && \text{on } \Gamma\times(0,T_0)\\ & u|_{t=0} = u_0 && \text{in } \Omega. \end{aligned}$$

Here and hereafter, Ω is a bounded domain in $\mathbf{R}^n$, $n \geqq 2$, whose boundary Γ is assumed to be a $C^{2,1}$ compact hypersurface, ν is the unit outer normal to $\Gamma, U(u,\pi), E(u)$ and $\tilde{E}(u)$ are nonlinear terms of the following forms:

$$U(u,\pi) = V_1\left(\int_0^t \nabla u\, d\tau\right)\nabla u + V_2\left(\int_0^t \nabla u\, d\tau\right)\pi$$
$$E(u) = V_3\left(\int_0^t \nabla u\, d\tau\right)\nabla u$$
$$\tilde{E}(u) = V_4\left(\int_0^t \nabla u\, d\tau\right)u$$

with some polynomials $V_j(\ \cdot\ )$ of $\int_0^t \nabla u\, d\tau$, $j = 1,2,3,4$, such as $V_j(0) = 0$. As a linearized problem of (3), we obtain the following problem:

$$(4)\quad \begin{aligned} & u_t - \mathrm{Div}\, S(u,\pi) = f && \text{in } \Omega\times(0,T_0)\\ & \mathrm{div}\, u = g = \mathrm{div}\,\tilde{g} && \text{in } \Omega\times(0,T_0)\\ & S(u,\pi)\nu|_\Gamma = h, \quad u|_{t=0} = u_0. \end{aligned}$$

Our purpose of this paper is to state L_p-L_q maximal regularity result for (4) and locally in time for any initial data and globally in time for small initial data unique existence theorems for (3). To state our theorems precisely, we now introduce the function spaces and some symbols. Let p and q denote exponents $\in [1,\infty]$, ℓ and m non-negative integers, I an interval of $\mathbf{R}$, D a domain in $\mathbf{R}^n$ and X a Banach space with norm $\|\cdot\|_X$. Let $L_q(D)$ and $W_q^m(D)$ denote the usual Lebesgue space and Sobolev space of order m on D and their norms are denoted by $\|\cdot\|_{L_q(D)}$ and $\|\cdot\|_{W_q^m(D)}$, respectively. Let $L_p(I,X)$ and $W_q^\ell(I,X)$ denote the usual Lebesgue space and Sobolev space of order m for the X-valued functions defined on I and their norms are denoted by $\|\cdot\|_{L_p(I,X)}$ and $\|\cdot\|_{W_q^\ell(I,X)}$, respectively. Set

$$W_{q,p}^{\ell,m}(D\times I) = L_p\left(I, W_q^\ell(D)\right)\cap W_p^m(I, L_q(D))$$
$$\|u\|_{W_{q,p}^{\ell,m}(D\times I)} = \|u\|_{L_p\left(I,W_q^\ell(D)\right)} + \|u\|_{W_p^m(I,L_q(D))}$$
$$W_{p,0}^1((0,T_0),X) = \left\{u\in W_p^1((-\infty,T_0),X) \mid u=0 \text{ for } t<0\right\}.$$

Given $\alpha\in\mathbf{R}$, we set

$$\langle D_t\rangle^\alpha u(t) = \mathcal{F}^{-1}\left[\left(1+s^2\right)^{\alpha/2}\mathcal{F}u(s)\right](t)$$
$$H_p^\alpha(\mathbf{R},X) = \{u\in L_p(\mathbf{R},X)\mid \langle D_t\rangle^\alpha u\in L_p(\mathbf{R},X)\}$$
$$\|u\|_{H_p^\alpha(\mathbf{R},X)} = \|\langle D_t\rangle^\alpha u\|_{L_p(\mathbf{R},X)} + \|u\|_{L_p(\mathbf{R},X)},$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively. Set

$$H_{q,p}^{1,1/2}(D\times\mathbf{R}) = H_p^{1/2}(\mathbf{R},L_q(D))\cap L_p\left(\mathbf{R},W_q^1(D)\right)$$
$$\|u\|_{H_{q,p}^{1,1/2}(D\times\mathbf{R})} = \|u\|_{H_p^{1/2}(\mathbf{R},L_q(D))} + \|u\|_{L_p\left(\mathbf{R},W_q^1(D)\right)}$$
$$H_{q,p,0}^{1,1/2}(D\times(0,\infty)) = \left\{u\in H_{q,p}^{1,1/2}(D\times(0,\infty)) \,\middle|\, u(t)=0 \text{ for } t<0\right\}.$$

Finally, given $0<T_0\leqq\infty$ we set

$$H_{q,p}^{1,1/2}(D\times(0,T_0)) = \left\{u \,\middle|\, \exists v\in H_{q,p}^{1,1/2}(D\times\mathbf{R}), u=v \text{ on } D\times(0,T_0)\right\}$$
$$\|u\|_{H_{q,p}^{1,1/2}(D\times(0,T_0))} = \inf\left\{\|v\|_{H_{q,p}^{1,1/2}(D\times\mathbf{R})} \,\middle|\, \begin{array}{l}\forall v\in H_{q,p}^{1,1/2}(D\times\mathbf{R}),\\ v=u \text{ on } D\times(0,T_0)\end{array}\right\}$$
$$H_{q,p,0}^{1,1/2}(D\times(0,T_0)) = \left\{u \,\middle|\, \begin{array}{l}\exists v\in H_{q,p,0}^{1,1/2}(D\times(0,\infty)),\\ u=v \text{ on } D\times(0,T_0)\end{array}\right\}$$
$$\|u\|_{H_{q,p,0}^{1,1/2}(D\times(0,T_0))} = \inf\left\{\|v\|_{H_{q,p}^{1,1/2}(D\times\mathbf{R})} \,\middle|\, \begin{array}{l}\forall v\in H_{q,p,0}^{1,1/2}(D\times(0,\infty)),\\ v=u \text{ on } D\times(0,T_0)\end{array}\right\}.$$

To state our main results concerning the unique existence of solutions to (4), we start with the analytic semigroup approach to the initial-boundary value problem:

$$(5)\quad \begin{aligned} & u_t - \mathrm{Div}\, S(u,\pi) = 0 && \text{in } \Omega\times(0,\infty)\\ & \mathrm{div}\, u = 0 && \text{in } \Omega\times(0,\infty)\\ & S(u,\pi)\nu|_\Gamma = 0, \quad u|_{t=0} = u_0. \end{aligned}$$

First of all we introduce the second Helmholtz decomposition corresponding to (5). Set

$$J_q(\Omega) = \{u = (u_1,\ldots,u_n)^* \in L_q(\Omega)^n \mid \mathrm{div}\, u = 0 \ \text{ in } \Omega\},$$
$$G_q(\Omega) = \left\{\nabla\pi \mid \pi\in W_q^1(\Omega),\ \pi|_\Gamma = 0\right\}.$$

Then, by Grubb and Solonnikov [2] we know that

$$L_q(\Omega)^n = J_q(\Omega) \oplus G_q(\Omega)$$

for $1 < q < \infty$, where $\oplus$ denotes the direct sum. Let P_q be the solenoidal projection: $L_q(\Omega)^n \to J_q(\Omega)$ along $G_q(\Omega)$. To introduce the generalized Stokes operator with Neumann boundary condition, we consider the resolvent problem corresponding to (5):

$$(6) \qquad \lambda v - \operatorname{Div} S(v,\theta) = P_q f, \ \operatorname{div} v = 0 \quad \text{in } \Omega, \qquad S(v,\theta)\nu|_\Gamma = 0.$$

Applying the divergence to (6) and multiplying the boundary condition by ν, we have

$$(7) \qquad \Delta\theta = 0 \quad \text{in } \Omega, \quad \theta|_\Gamma = \nu\cdot[D(v)\nu] - \operatorname{div} v|_\Gamma,$$

where we have used the facts that $\operatorname{div} u = 0$ in Ω and $\nu\cdot\nu = 1$ on Γ. We know that given $v \in W^2_q(\Omega)^n$ (7) admits a unique solution $\theta \in W^1_q(\Omega)$. From this point of view, let us define the map $K\colon W^2_q(\Omega) \to W^1_q(\Omega)$ by $\theta = K(v)$ for $v \in W^2_q(\Omega)$. By using this symbol, (6) is rewritten in the form:

$$(8) \qquad \lambda v - \operatorname{Div} S(v, K(v)) = P_q f \quad \text{in } \Omega, \qquad S(v,K(v))\nu|_\Gamma = 0.$$

We know that (6) and (8) are equivalent (cf. Grubb and Solonnikov [2]). We set

$$A_q v = -\operatorname{Div} S(v,K(v)) \quad \text{for } v \in \mathcal{D}(A_q)$$
$$\mathcal{D}(A_q) = \left\{ v \in J_q(\Omega)\cap W^2_q(\Omega)^n \;\middle|\; S(v,K(v))\nu|_\Gamma = 0 \right\}.$$

A_q with domain $\mathcal{D}(A_q)$ is our generalized Stokes operator with the Neumann boundary condition. From Grubb and Solonnikov [2] and Shibata and Shimizu [3] we know the following fact.

Theorem 1. *Let $1 < q < \infty$. Then, A_q generates an analytic semigroup $\{e^{-A_q t}\}_{t\geqq 0}$ on $J_q(\Omega)$.*

Now, we shall state our maximal regularity result for (4). The first one is concerned with the locally in time maximal regularity result for (4).

Theorem 2. *Let $1 < p, q < \infty$ and $T_0 > 0$. Set*

$$\mathcal{D}_{q,p}(\Omega) = [J_q(\Omega), \mathcal{D}(A_q)]_{1-1/p,p}$$

where $[\,\cdot\,,\cdot\,]_{\theta,p}$ denotes the real interpolation functor. If initial data u_0 and $f, g, \tilde g$ and h for (4) satisfy the condition:

$$u_0 \in \mathcal{D}_{q,p}(\Omega), \ f \in L_p((0,T_0), L_q(\Omega))^n$$
$$g \in L_p\left((0,T_0), W^1_q(\Omega)\right)$$
$$\tilde g \in W^1_{p,0}((0,T_0), L_q(\Omega))^n$$
$$h \in H^{1,1/2}_{q,p,0}(\Omega\times(0,T_0))^n$$

then (4) admits a unique solution

$$(u,\pi) \in W^{2,1}_{q,p}(\Omega\times(0,T_0))^n \times L_p\left((0,T_0), W^1_q(\Omega)\right)$$

which enjoys the estimate:

$$\begin{aligned}
&\|u\|_{W^{2,1}_{q,p}(\Omega\times(0,T_0))} + \|\pi\|_{L_p\left((0,T_0),W^1_q(\Omega)\right)} \\
&\leqq C(1+T_0)\Big\{\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} + \|f\|_{L_p((0,T_0),L_q(\Omega))} \\
&\qquad + \|h\|_{H^{1,1/2}_{q,p,0}(\Omega\times(0,T_0))} \\
&\qquad + \|g\|_{L_p\left((0,T_0),W^1_q(\Omega)\right)} \\
&\qquad + \|\tilde g\|_{W^1_p((0,T_0),L_q(\Omega))}\Big\},
\end{aligned}$$

where the constant C is independent of $T_0, u, \pi, f, g, \tilde g$ and h.

To state the globally in time maximal regularity result for (4), we have to introduce the rigid space $\mathcal{R}_d$ which is defined by

$$\mathcal{R}_d = \left\{ Ax+b \;\middle|\; \begin{array}{l} A\colon n\times n \text{ anti-symmetric matrix,} \\ b\in\mathbf{R}^n \end{array} \right\}.$$

In fact, we know that u satisfies the condition: $D(u) = 0$ if and only if $u \in \mathcal{R}_d$ and that if $u \in \mathcal{R}_d$, then $\operatorname{div} u = 0$. Therefore, if $u \in \mathcal{R}_d$, then u satisfies (4) with $f = g = \tilde g = h = 0$ and $u_0 = u$. In order for a solution (u,π) to (4) with $T_0 = \infty$ to be summable in $(0,\infty)$, we have to eliminate such solutions in $\mathcal{R}_d$. Let $p_\ell \in \mathcal{R}_d$, $\ell = 1,\dots,M$, be the basis of $\mathcal{R}_d$, which are normalized such as

$$(p_\ell, p_m)_\Omega = \delta_{\ell m}, \quad \ell, m = 1,\dots,M,$$

where $\delta_{\ell m}$ is the Kronecker delta symbol. We have the following theorem.

Theorem 3. *Let $1 < p, q < \infty$. Then, there exists a $\gamma_0 > 0$ such that if initial data u_0 and $f, g, \tilde g$ and h for (4) with $T_0 = \infty$ satisfy the following conditions:*

$$u_0 \in \mathcal{D}_{q,p}(\Omega), \ e^{\gamma t} f \in L_p((0,\infty), L_q(\Omega))^n$$
$$e^{\gamma t} g \in L_p\left((0,\infty), W^1_q(\Omega)\right)$$
$$e^{\gamma t}\tilde g \in W^1_{p,0}((0,\infty), L_q(\Omega))^n$$
$$e^{\gamma t} h \in H^{1,1/2}_{q,p,0}(\Omega\times(0,\infty))^n$$

for some $\gamma \in [0,\gamma_0]$ and

$$(u_0, p_\ell)_\Omega = 0$$
$$(f(\,\cdot\,,t), p_\ell)_\Omega + (h(\,\cdot\,,t), g_\ell)_\Gamma = 0$$

for a.e. $t > 0$ and $\ell = 1,\dots,M$, then (4) with $T_0 = \infty$ admits a unique solution

$$(u,\pi) \in W^{2,1}_{q,p}(\Omega \times (0,\infty))^n \times L_p\left((0,\infty), W^1_q(\Omega)\right)$$

which satisfies the estimates:

$$\begin{aligned}
&\|e^{\gamma t}u\|_{W^{2,1}_{q,p}(\Omega\times(0,\infty))} + \|e^{\gamma t}\pi\|_{L_p\left((0,\infty),W^1_q(\Omega)\right)} \\
&\leqq C\Big\{\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} + \|e^{\gamma t}f\|_{L_p((0,\infty),L_q(\Omega))} \\
&\qquad + \|e^{\gamma t}h\|_{H^{1,1/2}_{q,p,0}(\Omega\times(0,\infty))} \\
&\qquad + \|e^{\gamma t}g\|_{L_p\left((0,\infty)W^1_q(\Omega)\right)} \\
&\qquad + \|e^{\gamma t}\tilde{g}\|_{W^1_p((0,\infty),L_q(\Omega))}\Big\}
\end{aligned}$$

and the condition:

$$(u(\,\cdot\,,t),p_\ell)_\Omega = 0$$

for $t \geqq 0$ *and* $\ell = 1,\ldots,M$.

Roughly speaking, we can show our maximal regularity result as follows: First of all, we show the L_p-L_q maximal regularity of solutions to the model problems in the whole space and in the half-space by applying the Weis operator valued Fourier multiplier theorem ([5]) to the exact solution formulas, and therefore it is the key to show the $\mathcal{R}$ boundedness of the family of solution operators to the corresponding resolvent problem on $\mathcal{B}(L_q)$—the set of all bounded linear operators from L_q into itself (several techniques to show the $\mathcal{R}$ boundedness can be found in [1]). After such analysis for the model problems, using the usual localization procedure and estimating the perturbation terms by using the estimate: $\|e^{-A_q t}u_0\|_{W^1_q(\Omega)} \leqq Ct^{-1/2}e^{-ct}\|u_0\|_{L_q(\Omega)}$ ($c > 0$, u_0 being orthogonal to $\mathcal{R}_d$), we obtain the L_p-L_q maximal regularity result for (4) with $g = \tilde{g} = h = 0$. By using the solution to the Laplace equation with the zero Dirichlet boundary condition, we reduce the non-zero divergence condition to the divergence free case. Finally, non-homogeneous Neumann condition case is treated by using the solution to the dual problem with the homogeneous Neumann condition.

Our method can be applied to any initial boundary value problem for the equation of parabolic type with suitable boundary condition which generates an analytic semigroup, for example the Stokes equation with non-slip, slip or the Robin boundary conditions.

Finally, we shall state two unique existence theorems for (3) which can be proved by the contraction mapping principle based on Theorems 2 and 3.

Theorem 4. *Let* $2 < p < \infty$ *and* $n < q < \infty$. *Then, given* $u_0 \in \mathcal{D}_{q,p}(\Omega)$ *and* $f \in L_p((0,\infty),L_q(\mathbf{R}^n))^n$ *which has bounded derivatives with respect to* x *for each* t, *there exists a* $T_0 = T_0\big(\|u_0\|_{\mathcal{D}_{q,p}(\Omega)},\ \|f\|_{L_p((0,\infty),\,L_q(\mathbf{R}^n))},\ \sup_{t\geqq 0}\|\nabla f(\,\cdot\,,t)\|_{L_\infty(\mathbf{R}^n)}\big) > 0$ *such that* (3) *admits a unique solution*

$$(u,\pi) \in W^{2,1}_{q,p}(\Omega \times (0,T_0))^n \times L_p\left((0,T_0), W^1_q(\Omega)\right)$$

which satisfies the estimate:

$$\begin{aligned}
&\|u\|_{W^{2,1}_{q,p}(\Omega\times(0,T_0))} + \|\pi\|_{L_p\left((0,T_0),W^1_q(\Omega)\right)} \\
&\leqq C\left\{\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} + \|f\|_{L_p((0,T_0),L_q(\mathbf{R}^n))}\right\}.
\end{aligned}$$

Theorem 5. *Let* $2 < p < \infty$ *and* $n < q < \infty$. *Then, there exist positive numbers* ϵ *and* γ *such that if* $u_0 \in \mathcal{D}_{q,p}(\Omega)$, $\|u_0\|_{\mathcal{D}_{q,p}(\Omega)} \leqq \epsilon$ *and* $(u_0,p_\ell)_\Omega = 0$ *for* $\ell = 1,\ldots,M$, *then* (3) *with* $T_0 = \infty$ *and* $f = 0$ *admits a unique solution*

$$(u,\pi) \in W^{2,1}_{q,p}(\Omega \times (0,\infty))^n \times L_p\left((0,\infty), W^1_q(\Omega)\right)$$

which satisfies the estimate:

$$\begin{aligned}
&\|e^{\gamma t}u\|_{W^{2,1}_{q,p}(\Omega\times(0,\infty))} + \|e^{\gamma t}\pi\|_{L_p\left((0,\infty),W^1_q(\Omega)\right)} \\
&\leqq C\|u_0\|_{\mathcal{D}_{q,p}(\Omega)}
\end{aligned}$$

for some $\gamma > 0$ *and the condition*:

$$(u(\,\cdot\,,t),p_\ell)_\Omega = 0 \quad \text{for} \quad \ell = 1,\ldots,M \quad \text{and} \quad t \geqq 0.$$

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