Table of Quotient Curves of Modular Curves $X_0(N)$ with Genus 2

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1. Introduction. Let $X_0(N)$ be the modular curve corresponding to the modular group $\Gamma_0(N)$. All hyperelliptic curves $X_0(N)$ have been determined by Ogg [8], and their defining equations are given in [10]. Let $W(N)$ be the subgroup of $\mathrm{Aut}X_0(N)$ generated by all Atkin-Lehner's involutions ([1]), and let $W'(N)$ be a subgroup of $W(N)$. Then, by [2], we see that there are 138 cases in the range $N \leq 300$ such that the quotient curve $X' = X_0(N)/W'(N)$ is of genus two. For $N \geq 301$, we can find that there are only four such cases, see [4]. In this article, we shall give the table of defining equations of such X''s. We use the following

Theorem. *Let $S_2(N)$ be the space of cusp-forms of weight two on $\Gamma_0(N)$ and $S_2(N)^{W'(N)}$ be the fixed part of $S_2(N)$ by $W'(N)$. Assume that $X' = X_0(N)/W'(N)$ is of genus two, and $S_2(N)^{W'(N)}$ is spanned by*

$$f_1 = \sum_{n\geq 1} a_n q^n,\ f_2 = \sum_{n\geq 2} b_n q^n$$

with $a_n, b_n \in \boldsymbol{Z}$, $a_1 b_2 \neq 0$. Put $z = f_1/f_2$ and $w = (f_2/q)^{-1}(dz/dq)$. Then z and w generate the function field $\boldsymbol{Q}(X')$ of X' over $\boldsymbol{Q}$ and give the defining equation $w^2 = f(z)$ of X'.

Proof. See [7].

f_1 and f_2 are computed by using Brandt matrices and trace formulas of Hecke operators ([5], [9]).

2. Results. The following is the list of $X_0(N)/W'(N)$ with genus equal to 2. The fourth column gives $f(z)$ in the theorem. The sixth column gives the dimension of the subspace of $S_2(N)^{W'(N)}$ spanned by newforms, and the seventh column gives the field of Hecke eigenvalues if $S_2(N)^{W'(N)}$ is $\boldsymbol{Q}$-simple and spanned by newforms.

N	$p\mid N$	$W'(N)$	$f(z)$	disc(f)		
22	2,11	1	$(z^3+2z^2-4z+8)(z^3-2z^2+4z-4)$	$-2^{24}11^4$	0	
23	23	1	$(z^3-z+1)(z^3-8z^2+3z-7)$	$-2^{12}23^6$	2	$\boldsymbol{Q}(\sqrt{5})$
26	2,13	1	$z^6-8z^5+8z^4-18z^3+8z^2-8z+1$	$-2^{20}13^3$	2	
28	2,7	1	$(z^2+7)(z^2-z+2)(z^2+z+2)$	$2^{28}7^3$	0	
29	29	1	$z^6-4z^5-12z^4+2z^3+8z^2+8z-7$	$-2^{12}29^5$	2	$\boldsymbol{Q}(\sqrt{2})$
30	2,3,5	$\langle W_2\rangle$	$(z^2-z+1)(z^2+3z+1)(z^2-6z+1)$	$2^{13}3^9 5^5$	1	
		$\langle W_3\rangle$	$z(z+3)(z^2-z+4)(z^2-2z+5)$	$-2^{24}3^3 5^5$	0	
		$\langle W_{10}\rangle$	$(z^2-z-1)(z^2+2z-7)(z^2+3z-9)$	$-2^{13}3^6 5^2$	1	
31	31	1	$(z^3-2z^2-z+3)(z^3-6z^2-5z-1)$	$-2^{12}31^4$	2	$\boldsymbol{Q}(\sqrt{5})$
33	3,11	$\langle W_3\rangle$	$(z-1)(z^2-4z-8)(z^3+z^2+3z-1)$	$2^{12}3\cdot 11^9$	1	
35	5,7	$\langle W_7\rangle$	$(z+1)(z^2+4)(z^3-5z^2+3z-19)$	$-2^{12}5^7 7^3$	2	$\boldsymbol{Q}(\sqrt{17})$
37	37	1	$z^6+8z^5-20z^4+28z^3-24z^2+12z-4$	$-2^{12}37^3$	2	
38	2,19	$\langle W_2\rangle$	$(z^3-5z^2-4)(z^3+z^2-z+3)$	$-2^{15}19^6$	1	
39	3,13	$\langle W_{13}\rangle$	$(z-1)(z+3)(z^2-5z+3)(z^2+3z-1)$	$-2^{12}3^{10}13^2$	2	$\boldsymbol{Q}(\sqrt{2})$
40	2,5	$\langle W_8\rangle$	$(z^2+4z-4)(z^4+4z^2-8z+4)$	$-2^{25}5^5$	1	
		$\langle W_5\rangle$	$(z^2+4)(z^4+12z^2+16)$	$2^{36}5^2$	0	
42	2,3,7	$\langle W_3\rangle$	$(z^2-5z+1)(z^4+z^3+4z^2+z+1)$	$-2^{25}3\cdot 7^3$	1	
		$\langle W_6\rangle$	$(z^2+z-5)(z^4-z^3-2z^2-5z+11)$	$-2^{15}3\cdot 7^3$	0	
		$\langle W_{21}\rangle$	$(z^2+z+1)(z^4+7z^3+16z^2+7z+1)$	$2^{19}3\cdot 7^2$	1	
		$\langle W_{42}\rangle$	$(z^2-5z+7)(z^4-7z^3+22z^2-35z+23)$	$2^{13}3\cdot 7^2$	0	

N	$p\mid N$	$W'(N)$	$f(z)$	$\mathrm{disc}(f)$		
44	2,11	$\langle W_4\rangle$	$(z^3+4z^2-16)(z^3+8z^2+24z+28)$	$-2^{24}11^4$	0	
46	2,23	$\langle W_{46}\rangle$	$(z^3-4z^2+z+7)(z^3-4z^2+5z-1)$	$-2^{12}23^2$	0	
48	2,3	$\langle W_{16}\rangle$	$(z^2-2z+2)(z^2+2z-2)(z^2+4z-4)$	$2^{25}3^5$	1	
		$\langle W_3\rangle$	$(z^2+4)(z^2-2z+4)(z^2+2z+4)$	$2^{36}3^2$	0	
50	2,5	1	$z^6-4z^5-10z^3-4z+1$	$-2^{16}5^5$	2	
52	2,13	$\langle W_4\rangle$	$z^6+8z^5+8z^4+18z^3+8z^2+8z+1$	$-2^{20}13^3$	0	
		$\langle W_{52}\rangle$	$z^6-8z^5+28z^4-58z^3+84z^2-84z+41$	$2^{17}13^2$	0	
54	2,3	$\langle W_2\rangle$	$(z^3-3z^2-4)(z^3+3z^2+3z+5)$	$-2^{13}3^{14}$	1	
57	3,19	$\langle W_3\rangle$	$z^6+4z^5-20z^4+44z^3-52z^2+36z-12$	$-2^{12}3\cdot 19^3$	1	
		$\langle W_{57}\rangle$	$z^6-8z^5+28z^4-52z^3+56z^2-36z+12$	$2^{12}3\cdot 19^2$	1	
58	2,29	$\langle W_{29}\rangle$	$z^6+4z^5+16z^4+22z^3+16z^2+4z+1$	$2^{20}29^2$	2	
60	2,3,5	$\langle W_{20}\rangle$	$(z^2+z+1)(z^4+3z^3+8z^2+3z+1)$	$2^{14}3^3 5^2$	0	
		$\langle W_4, W_3\rangle$	$z(z-3)(z^2+z+4)(z^2+2z+5)$	$-2^{24}3^3 5^5$	0	
		$\langle W_5, W_{12}\rangle$	$(z-1)(z-2)(z^2-3z+6)(z^2-2z+5)$	$-2^{20}3\cdot 5^3$	0	
62	2,31	$\langle W_{62}\rangle$	$(z^3-2z^2+z+1)(z^3-6z^2+13z-11)$	$-2^{12}31^2$	0	
66	2,3,11	$\langle W_{11}\rangle$	$(z^2-z+1)(z^4-3z^3-4z^2-3z+1)$	$-2^{17}3^3 11^2$	2	
		$\langle W_2, W_3\rangle$	$(z+2)(z^2-3z-6)(z^3-3z^2-z+7)$	$2^{12}3\cdot 11^4$	0	
		$\langle W_3, W_{22}\rangle$	$z(z-3)(z-4)(z^3-9z^2+23z-11)$	$2^{12}3^2 11^3$	0	
		$\langle W_6, W_{22}\rangle$	$z(z^2+z-8)(z^3+3z^2-z+1)$	$2^{14}3\cdot 11^2$	1	
67	67	$W(67)$	$z^6-4z^5+6z^4-6z^3+9z^2-14z+9$	$-2^{12}67^2$	2	$\boldsymbol{Q}(\sqrt{5})$
68	2,17	$\langle W_{68}\rangle$	$(z-1)(4z^4-23z^3+55z^2-64z+32)$	$2^{13}17^2$	0	
69	3,23	$\langle W_{69}\rangle$	$(z^3-6z^2+11z-5)(z^3-2z^2+3z-1)$	$-2^{12}23^2$	0	
70	2,5,7	$\langle W_{35}\rangle$	$(z^2+3z+1)(z^4+z^3+4z^2+z+1)$	$-2^{17}5\cdot 7^2$	1	
		$\langle W_2, W_5\rangle$	$(z-2)(z^2+z+2)(z^3+5z^2+7z+7)$	$-2^{14}5\cdot 7^6$	0	
		$\langle W_2, W_7\rangle$	$z(z+1)(z-4)(z^3-z^2-z+5)$	$2^{12}5^5 7^5$	0	
		$\langle W_7, W_{10}\rangle$	$z(z^2-3z+4)(z^3-z^2-z+5)$	$-2^{12}5^3 7^2$	0	
72	2,3	$\langle W_8\rangle$	$(z^2+2z-2)(z^4-2z^3+6z^2-8z+4)$	$-2^{18}3^8$	1	
		$\langle W_{72}\rangle$	$(z^2-2z+2)(z^4-6z^3+18z^2-24z+12)$	$2^{18}3^3$	0	
73	73	$W(73)$	$z^6+8z^5+26z^4+50z^3+61z^2+38z+9$	$-2^{12}73^2$	2	$\boldsymbol{Q}(\sqrt{5})$
74	2,37	$\langle W_{74}\rangle$	$z^6-8z^5+24z^4-36z^3+28z^2-12z+4$	$-2^{12}37^2$	0	
77	7,11	$\langle W_{77}\rangle$	$z^6-4z^5+12z^4-20z^3+20z^2-12z+4$	$2^{12}7\cdot 11^2$	1	
78	2,3,13	$\langle W_{39}\rangle$	$z^6+4z^5+8z^4+6z^3+8z^2+4z+1$	$2^{17}13^2$	0	
		$\langle W_2, W_{13}\rangle$	$(z-3)(z+1)(z^2+z-3)(z^2+z+1)$	$2^{12}3^7 13^3$	0	
80	2,5	$\langle W_{80}\rangle$	$(z^2-2z+2)(z^4-2z^3-2z^2+8z-4)$	$-2^{18}5^2$	0	
84	2,3,7	$\langle W_4, W_3\rangle$	$(z^2+5z+1)(z^4-z^3+4z^2-z+1)$	$-2^{25}3\cdot 7^3$	0	
		$\langle W_4, W_{21}\rangle$	$(z^2-z+1)(z^4-7z^3+16z^2-7z+1)$	$2^{19}3\cdot 7^2$	0	
		$\langle W_3, W_{28}\rangle$	$(z^2-3z+3)(z^4-z^3-2z^2-5z+11)$	$2^{19}3\cdot 7^2$	0	
		$\langle W_7, W_{12}\rangle$	$z(z^2-3z+4)(4z^2-3z+1)$	$2^{14}3^4 7^2$	0	
85	5,17	$W(85)$	$(z^2-2z+5)(z^4-2z^3+3z^2-6z+5)$	$2^{12}5^4 17^3$	2	$\boldsymbol{Q}(\sqrt{2})$
87	3,29	$\langle W_{29}\rangle$	$(z^3-2z^2-z-1)(z^3+2z^2+3z+3)$	$-2^{12}3^4 29^2$	2	$\boldsymbol{Q}(\sqrt{5})$
		$\langle W_{87}\rangle$	$z^6-4z^5+12z^4-22z^3+32z^2-28z+17$	$2^{12}29^2$	0	
88	2,11	$W(88)$	$(z^3-6z^2+8z-4)(z^3-2z^2+4z-4)$	$-2^{24}11^2$	1	
90	2,3,5	$\langle W_9, W_5\rangle$	$(z^2-z+1)(z^4+5z^3+12z^2+5z+1)$	$2^{21}3^4 5^2$	1	
		$\langle W_2, W_{45}\rangle$	$(z^2-z+1)(z^4-3z^3+8z^2-3z+1)$	$2^{14}3^3 5^2$	0	

N	$p\mid N$	$W'(N)$	$f(z)$	disc(f)		
		$\langle W_9, W_{10}\rangle$	$(z^2-3z+3)(z^4-5z^3+14z^2-25z+19)$	$2^{14}3^3 5^2$	0	
		$\langle W_5, W_{18}\rangle$	$(z-1)(z^2-z+4)(4z^2-7z+4)$	$2^{12}3^8 5^2$	0	
91	7,13	$\langle W_{91}\rangle$	$z^6-4z^5+4z^4-4z^3+12z^2-12z+4$	$-2^{12}7^2 13^2$	2	
93	3,31	$W(93)$	$(z^3-2z^2-z+3)(z^3+2z^2-5z+3)$	$-2^{12}3^4 31^2$	2	$\boldsymbol{Q}(\sqrt{5})$
98	2,7	$\langle W_{98}\rangle$	$(z-1)(4z^4-11z^3+19z^2-16z+8)$	$2^{11}7^3$	0	
100	2,5	$\langle W_4\rangle$	$z^6+4z^5+10z^3+4z+1$	$-2^{16}5^5$	0	
102	2,3,17	$\langle W_3, W_{17}\rangle$	$(z^2-z+1)(z^4+5z^3+4z^2+5z+1)$	$-2^{18}3\cdot 17^2$	1	
		$\langle W_2, W_{51}\rangle$	$(z^2-z+1)(z^4-3z^3-3z+1)$	$-2^{13}3\cdot 17^2$	1	
103	103	$W(103)$	$z^6-10z^4+22z^3-19z^2+6z+1$	$-2^{12}103^2$	2	$\boldsymbol{Q}(\sqrt{5})$
104	2,13	$W(104)$	$(z-1)(z^2-2z+5)(z^3-z^2+4)$	$-2^{21}13^3$	0	
106	2,53	$W(106)$	$z^6-4z^5+4z^4+2z^3+4z^2-4z+1$	$2^{14}53^2$	1	
107	107	$W(107)$	$z^6-4z^5+10z^4-18z^3+17z^2-10z+1$	$-2^{12}107^2$	2	$\boldsymbol{Q}(\sqrt{5})$
110	2,5,11	$\langle W_{10}, W_{22}\rangle$	$(z^3-z^2-8)(z^3+z^2+3z-1)$	$-2^{13}5\cdot 11^2$	1	
111	3,37	$\langle W_{111}\rangle$	$z^6-4z^5+4z^4+4z^3-12z^2+12z-4$	$-2^{12}37^2$	0	
112	2,7	$W(112)$	$(z^2-2z+2)(z^4-2z^3+10z^2-16z+8)$	$2^{25}7^2$	1	
114	2,3,19	$\langle W_3, W_{38}\rangle$	$z^6-4z^5+8z^4-12z^3+16z^2-12z+4$	$2^{12}3\cdot 19^2$	0	
115	5,23	$W(115)$	$(z^3-2z^2+3z-1)(z^3+2z^2-9z+7)$	$-2^{12}5^4 23^2$	2	$\boldsymbol{Q}(\sqrt{5})$
116	2,29	$W(116)$	$z^6-4z^5+16z^4-22z^3+16z^2-4z+1$	$2^{20}29^2$	0	
117	3,13	$W(117)$	$(z^2-z+1)(z^4+z^3+4z^2-3z+9)$	$2^{12}3^5 13^2$	0	
120	2,3,5	$\langle W_8, W_{15}\rangle$	$(z^2-2z+2)(z^4-6z^3+10z^2-8z+4)$	$-2^{18}5^2$	0	
		$\langle W_{24}, W_{40}\rangle$	$(z^2+2z-2)(z^4-2z^3-2z^2+8z-4)$	$2^{18}3\cdot 5^2$	1	
121	11	$W(121)$	$z^6-4z^4+4z^3+8z^2-12z+4$	$2^{12}11^3$	1	
122	2,61	$W(122)$	$z^6+4z^4-6z^3+4z^2+1$	$2^{14}61^2$	1	
125	5	$W(125)$	$z^6-4z^5+10z^4-10z^3+5z^2+2z-3$	$-2^{12}5^6$	2	$\boldsymbol{Q}(\sqrt{5})$
126	2,3,7	$\langle W_2, W_{63}\rangle$	$(z^2+z+1)(z^4-z^3+4z^2-z+1)$	$2^{13}3\cdot 7^2$	0	
		$\langle W_{18}, W_{14}\rangle$	$(z-1)(4z^4+z^3+3z^2-8z+4)$	$2^{11}3^6 7^2$	1	
129	3,43	$W(129)$	$z^6-4z^5-4z^4+12z^3+4z^2-12z+4$	$-2^{12}3^2 43^2$	1	
130	2,5,13	$\langle W_{10}, W_{26}\rangle$	$z(4z^4+17z^3+22z^2+17z+4)$	$-2^{14}5^2 13^2$	1	
132	2,3,11	$\langle W_4, W_{11}\rangle$	$(z^2+z+1)(z^4+3z^3-4z^2+3z+1)$	$-2^{17}3^3 11^2$	0	
133	7,19	$W(133)$	$z^6+4z^5-18z^4+26z^3-15z^2+2z+1$	$-2^{12}7^2 19^2$	2	$\boldsymbol{Q}(\sqrt{5})$
134	2,67	$W(134)$	$z^6-4z^5+2z^4-2z^3+z^2+2z+1$	$-2^{12}67^2$	0	
135	3,5	$W(135)$	$z(z^2-z+4)(z^3-3z^2+3z-5)$	$-2^{12}3^8 5^3$	1	
138	2,3,23	$\langle W_3, W_{23}\rangle$	$(z^2+z+1)(z^4-z^3+8z^2-z+1)$	$2^{17}3^2 23^2$	2	
		$\langle W_{23}, W_6\rangle$	$z(4z^4-3z^3+10z^2-3z+4)$	$2^{11}3^2 23^2$	1	
140	2,5,7	$\langle W_4, W_{35}\rangle$	$(z^2-3z+1)(z^4-z^3+4z^2-z+1)$	$-2^{17}5\cdot 7^2$	0	
142	2,71	$\langle W_{71}\rangle$	$z^6+4z^5-2z^3+4z+1$	$-2^{15}71^2$	2	
143	11,13	$\langle W_{143}\rangle$	$z^6-4z^4+12z^3-16z^2+12z-4$	$-2^{12}11^2 13$	1	
146	2,73	$W(146)$	$z^6-4z^5+2z^4+6z^3+z^2+2z+1$	$-2^{12}73^2$	0	
147	3,7	$W(147)$	$(z^2-z+1)(z^4+z^3-4z^2-3z+9)$	$2^{12}3^3 7^5$	2	$\boldsymbol{Q}(\sqrt{2})$
150	2,3,5	$\langle W_6, W_{50}\rangle$	$z(4z^4+9z^3+14z^2+9z+4)$	$2^{11}3^2 5^3$	1	
153	3,17	$W(153)$	$z^6-4z^5+4z^4+12z^3-36z^2+36z-12$	$-2^{12}3^4 17^2$	1	
154	2,7,11	$W(154)$	$(z-2)(z^2+z+2)(z^3-3z^2-z-1)$	$-2^{14}7^3 11^3$	1	
156	2,3,13	$\langle W_4, W_{39}\rangle$	$z^6-2z^5+3z^4-6z^3+13z^2-12z+4$	$2^{17}13^2$	0	
158	2,79	$W(158)$	$z^6-4z^4+2z^3-4z^2+1$	$2^{13}79^2$	1	

N	$p\mid N$	$W'(N)$	$f(z)$	$\mathrm{disc}(f)$		
161	7,23	$W(161)$	$(z^3-2z^2+3z-1)(z^3+2z^2+3z-5)$	$-2^{12}7^4 23^2$	2	$\boldsymbol{Q}(\sqrt{5})$
165	3,5,11	$W(165)$	$(z-1)(z+3)(z^2-z-1)(z^2-z+3)$	$2^{12}3^4 5^3 11^3$	2	$\boldsymbol{Q}(\sqrt{2})$
166	2,83	$W(166)$	$(z^3-2z^2+2z+1)(z^3+2z^2-2z+1)$	$-2^{14}83^2$	1	
167	167	$W(167)$	$z^6-4z^5+2z^4-2z^3-3z^2+2z-3$	$-2^{12}167^2$	2	$\boldsymbol{Q}(\sqrt{5})$
168	2,3,7	$W(168)$	$(z-1)(z+2)(z^2+3)(z^2-z+2)$	$-2^{18}3^3 7^3$	0	
170	2,5,17	$W(170)$	$(z^2-z-1)(z^4-3z^3+6z^2+5z-1)$	$2^{12}5^3 17^2$	0	
177	3,59	$W(177)$	$z^6+2z^4-6z^3+5z^2-6z+1$	$-2^{12}3^2 59^2$	2	$\boldsymbol{Q}(\sqrt{5})$
180	2,3,5	$W(180)$	$(z^2+z+1)(z^4-5z^3+12z^2-5z+1)$	$2^{21}3^4 5^2$	0	
184	2,23	$W(184)$	$z^6-4z^5+8z^4-16z^3+28z^2-24z+8$	$2^{18}23^2$	1	
186	2,3,31	$W(186)$	$(z^3-2z^2+z+1)(z^3+2z^2+5z+1)$	$-2^{12}3^4 31^2$	0	
190	2,5,19	$\langle W_5, W_{19}\rangle$	$z^6+8z^4-2z^3+8z^2+1$	$2^{16}5\cdot 19^2$	1	
191	191	$W(191)$	$z^6+2z^4+2z^3+5z^2-6z+1$	$-2^{12}191^2$	2	$\boldsymbol{Q}(\sqrt{5})$
198	2,3,11	$W(198)$	$(z^2-z+1)(z^4+5z^3+5z+1)$	$-2^{13}3^4 11^2$	0	
204	2,3,17	$W(204)$	$(z^2+z+1)(z^4-5z^3+4z^2-5z+1)$	$-2^{18}3\cdot 17^2$	0	
205	5,41	$W(205)$	$z^6+2z^4+10z^3+5z^2-6z+1$	$-2^{12}5^2 41^2$	2	$\boldsymbol{Q}(\sqrt{5})$
206	2,103	$W(206)$	$z^6+2z^4+2z^3+5z^2+6z+1$	$-2^{12}103^2$	0	
209	11,19	$W(209)$	$z^6-4z^5+8z^4-8z^3+8z^2+4z+4$	$2^{12}11^2 19^3$	2	$\boldsymbol{Q}(\sqrt{2})$
210	2,3,5,7	$\langle W_6, W_{10}, W_{14}\rangle$	$z(4z^4-3z^3+2z^2-3z+4)$	$2^{12}3^2 5^2 7^2$	1	
213	3,71	$W(213)$	$z^6+2z^4+2z^3-7z^2+6z-3$	$-2^{12}3^2 71^2$	2	$\boldsymbol{Q}(\sqrt{5})$
215	5,43	$W(215)$	$z^6+4z^5-12z^4+20z^3-20z^2+12z-4$	$-2^{12}5^2 43^2$	1	
221	13,17	$W(221)$	$z^6+4z^5+2z^4+6z^3+z^2-2z+1$	$-2^{12}13^2 17^2$	2	$\boldsymbol{Q}(\sqrt{5})$
230	2,5,23	$W(230)$	$(z^3-2z^2+5z+1)(z^3+2z^2+z+1)$	$-2^{12}5^4 23^2$	0	
255	3,5,17	$W(255)$	$(z^2+z-1)(z^4-5z^3-6z^2+3z-1)$	$2^{12}5^3 17^2$	0	
266	2,7,19	$W(266)$	$z^6+4z^5+10z^4+14z^3+17z^2+10z+1$	$-2^{12}7^2 19^2$	0	
276	2,3,23	$W(276)$	$(z^2-z+1)(z^4+z^3+8z^2+z+1)$	$2^{17}3^2 23^2$	0	
284	2,71	$W(284)$	$z^6-4z^5+2z^3-4z+1$	$-2^{15}71^2$	0	
285	3,5,19	$W(285)$	$z(z^2+z+4)(z^3-z^2-z-3)$	$-2^{12}3^3 5\cdot 19^3$	1	
286	2,11,13	$W(286)$	$(z^3-z^2+3z+1)(z^3+z^2-4)$	$-2^{13}11^3 13^3$	1	
287	7,41	$W(287)$	$z^6-4z^5+2z^4+6z^3-15z^2+14z-7$	$-2^{12}7^2 41^2$	2	$\boldsymbol{Q}(\sqrt{5})$
299	13,23	$W(299)$	$z^6-4z^5+6z^4+6z^3-7z^2-10z-3$	$-2^{12}13^2 23^2$	2	$\boldsymbol{Q}(\sqrt{5})$
330	2,3,5,11	$W(330)$	$(z^2+z+1)(z^4-z^3+8z^2+3z+9)$	$2^{12}3^3 5^2 11^2$	0	
357	3,7,17	$W(357)$	$z^6+8z^4-8z^3+20z^2-12z+12$	$2^{12}3^2 7^3 17^2$	2	$\boldsymbol{Q}(\sqrt{2})$
380	2,5,19	$W(380)$	$z^6+8z^4+2z^3+8z^2+1$	$2^{16}5\cdot 19^2$	0	
390	2,3,5,13	$W(390)$	$(z^2-z+1)(z^4+5z^3-8z^2+5z+1)$	$-2^{14}3\cdot 5^2 13^2$	1	

3. Remarks. (1) Defining equations of hyperelliptic curves of genus two of type $X_0(p)$, $X_0^*(p) = X_0(p)/\langle W_p\rangle$ where p is a prime have been given in [3] and [7], and M. Shimura has calculated in [10] defining equations of *all* $X_0(N)$ with genus $2 \le g \le 6$.

(2) Hyperelliptic curves of type $X_1(N)$ have been determined by [6]. There are only three values of N for which $X_1(N)$ is hyperelliptic, namely $N = 13$, 16 and 18, and all of these are genus two. We have also calculated their defining equations by the same method as in the case $X_0(N)/W'(N)$:

$$X_1(13): w^2 = z^6 - 8z^5 + 26z^4 - 46z^3 + 53z^2 - 42z + 17,$$

$$X_1(16): w^2 = (z-1)(z+1)(z^2+1) \times (z^2+2z-1),$$

$$X_1(18): w^2 = z^6 + 8z^5 + 30z^4 + 70z^3 + 105z^2 + 90z + 33.$$

References

[1] A. O. L. Atkin and J. Lehner: Hecke operators on $\Gamma_0(m)$. Math. Ann., **185**, 134–160 (1970).

[2] A. O. L. Atkin and D. J. Tingley: Modular functions of one variable. IV. Lect. Notes in Math., no. 476, Springer-Verlag, Berlin, New York, Table 5, pp. 135–141 (1975).

[3] R. Fricke: Die Elliptischen Funktionen und ihre Anwendungen. Leipzig and Berlin (1916).

[4] Y. Hasegawa and K. Hashimoto: Hyperelliptic modular curves $X_0^*(N)$ with square-free levels (submitted).

[5] H. Hijikata: Explicit formula of the traces of Hecke operators for $\Gamma_0(N)$. J. Math. Soc. Japan, **26**, 56–82 (1974).

[6] N. Ishii and F. Momose: Hyperelliptic modular curves. Tsukuba J. Math., **15**, 413–423 (1991).

[7] N. Murabayashi: On normal forms of modular curves of genus 2. Osaka J. Math., **29**, 405–418 (1992).

[8] A. P. Ogg: Hyperelliptic modular curves. Bull. Soc. Math. France, **102**, 449-462 (1974).

[9] A. Pizer: An algorithm for computing modular forms on $\Gamma_0(N)$. J. Algebra, **64**, 340-390 (1980).

[10] M. Shimura: Defining equations of modular curves $X_0(N)$ (to appear in Tokyo J. Math.).