19. *A Continuation Principle for the 3-D Euler Equations for Incompressible Fluids in a Bounded Domain*

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1. In this paper we study the Euler equations for ideal incompressible fluids in a bounded domain Ω in $\boldsymbol{R}^3$:

(1) $$u_t + u\cdot\nabla u + \nabla p = 0, \quad \nabla\cdot u = 0 \text{ for } t \geq 0,\ x \in \Omega,$$

(2) $$u\cdot n = 0 \quad \text{for} \quad t \geq 0,\ x \in \Gamma.$$

Here the boundary Γ of Ω is assumed to be of class C^∞; t and x are time and space variables; $u = u(t, x) = (u_1, u_2, u_3)$ is the velocity and $p = p(t, x)$ is the pressure; $n = n(x) = (n_1, n_2, n_3)$ is the unit outward normal at $x \in \Gamma$; we write $u_t = \partial u/\partial t$, $\partial_i = \partial/\partial x^i$ for $i = 1, 2, 3$, $\nabla = (\partial_1, \partial_2, \partial_3)$ and $u\cdot\nabla = \sum_{i=1}^3 u_i\partial_i$.

Let $s \geq 0$ be an integer. We denote by $H^s(\Omega\,;\boldsymbol{R}^3)$ the usual Sobolev space of order s on Ω taking values in $\boldsymbol{R}^3$. The norm is defined by $\| u \|_s^2 = \sum_{|\alpha|\leq s} | \partial^\alpha u |_{L^2(\Omega)}^2$, where $\partial^\alpha = \partial^{|\alpha|}/\partial_1^{\alpha_1}\partial_2^{\alpha_2}\partial_3^{\alpha_3}$ with $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. For $0 < T < \infty$, we put

$$X_s(T) = C^0([0, T]\,; H^s(\Omega\,;\boldsymbol{R}^3)) \cap C^1([0, T]\,; H^{s-1}(\Omega\,;\boldsymbol{R}^3)).$$

Now we state our main

Theorem. *Let $s > 2$ be an integer. Suppose that u is a solution of* (1), (2) *belonging to $X_s(T')$ for any $T' < T < \infty$ such that $\| u(t) \|_s \uparrow \infty$ as $t \uparrow T$. Then*

(3) $$\int_0^t | \operatorname{rot} u(\tau) |_{L^\infty(\Omega)} d\tau \uparrow \infty \text{ as } t \uparrow T.$$

This theorem is an immediate consequence of the local in time existence theorem for the initial boundary value problem (1), (2) with the initial data $u^0 \in H^s(\Omega\,;\boldsymbol{R}^3)$ satisfying $\nabla\cdot u^0 = 0$ in Ω, $u^0\cdot n = 0$ on Γ (see [3,6]), and the following new estimate for a smooth solution u of (1), (2) such that $u \in X_s(T)$ with $s > 2$: There exists a nondecreasing continuous function $F(t, x, y) \geq 0$ for $t \geq 0$, $x \geq 0$, $y \geq 0$, satisfying the estimate

(4) $$\| u(t) \|_s \leq F(t, \| u(0) \|_s, \int_0^t | \operatorname{rot} u(\tau) |_{L^\infty(\Omega)} d\tau) \quad \text{for } t \in [0, T].$$

In the sequel, C is a constant which might change line by line and $u(t, x)$ is always a smooth solution of (1), (2) in the sense mentioned above.

Such a link that exists between the accumulation of the vorticity and the possible breakdown of smooth solutions for the Euler equations was shown by Beale-Kato-Majda [2] for the motion of fluids in the entire space $\boldsymbol{R}^3$.

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When Ω is a bounded domain, the arguments become more involved, because of the appearance of the boundary. Recently Ferrari [5] discussed this link for simply connected domains in $\boldsymbol{R}^3$ by utilizing the Green's matrix of Solonnikov [8] for the boundary value problem of an elliptic system introduced by himself.

In order to state the reasoning in a clear-cut way, we use the theory of harmonic integrals. The crucial estimate (15) is shown by considering the generalized Biot-Savart law (6) and the representation of a parametrix of Laplacian on 2-forms on Ω. To derive this representation, we also apply the result of [8].

2. Let $H^s(\Omega\,;\Lambda^\ell)$ be the Hilbert space of ℓ-forms Λ^ℓ on Ω with the usual norm of $H^s(\Omega)$. In what follows we identify a vector field u and the vorticity rot $u=(w_1, w_2, w_3)$ on Ω with a 1-form $u_1dx^1+u_2dx^2+u_3dx^3$ and a 2-form $w_1dx^2\wedge dx^3+w_2dx^3\wedge dx^1+w_3dx^1\wedge dx^2$ on Ω. Here the canonical metric of $\boldsymbol{R}^3$ is induced into Ω. Next let d_ℓ, δ_ℓ, and $*$ denote the exterior derivative on ℓ-forms, the codifferential operator of $d_{\ell-1}$, and the Hodge star operator, respectively. ι is the inclusion map $\Gamma\rightarrow\bar{\Omega}$ and ι^* denotes the induced map of ι. In general, for a differentiable mapping Φ, Φ^* denotes its induced map. (For definitions in the above, see [9].) Then Laplacian Δ_ℓ on ℓ-forms and the space of harmonic ℓ-forms $\mathscr{H}_\ell(\Omega)$ on Ω are defined by

$$\Delta_\ell=d_{\ell-1}\delta_\ell+\delta_{\ell+1}d_\ell,$$
$$\mathscr{H}_\ell(\Omega)=\{w\in H^1(\Omega\,;\Lambda^\ell)\mid d_\ell w=0,\ \delta_\ell w=0 \text{ on } \Omega,\ \iota^*(*w)=0\}.$$

We summarize the statement of the decomposition theorem on Ω as follows: (See Theorems 7.7.1-7.7.4 in [7], Theorem 10.5 in [1] with the fact remarked after (8). See also [4].)

i) For $\ell=1,2$, Laplacian Δ_ℓ on ℓ-forms with the domain

$$D(\Delta_\ell)=\{w\in H^2(\Omega\,;\Lambda^\ell)\mid \iota^*(*w)=\iota^*(*d_\ell w)=0\},$$

has the kernel and the cokernel equal to $\mathscr{H}_\ell(\Omega)$, which is a finite dimensional subspace included in $C^\infty(\bar{\Omega}\,;\Lambda^\ell)$.

ii) For $\ell=1,2$, the space $H^s(\Omega\,;\Lambda^\ell)$ is decomposed as

$$H^s(\Omega\,;\Lambda^\ell)=\mathscr{H}_\ell(\Omega)\oplus\delta_{\ell+1}d_\ell\Delta_\ell^{-1}(\mathscr{H}_\ell(\Omega)^\perp)\oplus d_{\ell-1}\delta_\ell\Delta_\ell^{-1}(\mathscr{H}_\ell(\Omega)^\perp). \tag{5}$$

Here Δ_ℓ^{-1} is the inverse of Δ_ℓ on $\mathscr{H}_\ell(\Omega)^\perp$ which is the L^2-orthogonal complement of $\mathscr{H}_\ell(\Omega)$ in $H^s(\Omega\,;\Lambda^\ell)$, and all subspaces on the right side of (5) are L^2-orthogonal to each other.

iii) Since $u(t,x)$ is L^2-orthogonal to $d_0\delta_1\Delta_1^{-1}(\mathscr{H}_1(\Omega)^\perp)$ in (5), we obtain from ii)

$$u(t,x)=\sum_{i=1}^{R}\lambda_i(t)a_i(x)+\delta_2\Delta_2^{-1}d_1u(t,x),\ t\in[0,T],\ x\in\Omega, \tag{6}$$

where $R=\dim\mathscr{H}_1(\Omega)$, $\{a_i(x)\}_{i=1}^R\subset C^\infty(\bar{\Omega})$ is an L^2-orthogonal basis of $\mathscr{H}_1(\Omega)$, and $\lambda_i(t)=(u(t,x), a_i(x))_{L^2(\Omega)}$, $1\le i\le R$. Here we used the fact that $d_1\Delta_1^{-1}=\Delta_2^{-1}d_1$ (see p. 547 in [4]) and $d_1u(t,\cdot)\in\mathscr{H}_2(\Omega)^\perp$.

3. We use an appropriate parametrix of Δ_2. Choose an open cover of

Ω, $\{\mathcal{O}^\gamma\}_{\gamma=0}^k$, such that $\cup_{\gamma=0}^k \mathcal{O}^\gamma = \Omega$, $\mathcal{O}^0 \subset\subset \Omega$, $\overline{\mathcal{O}^\gamma} \cap \Gamma \neq \phi$, $1 \leq \gamma \leq k$, and each $\mathcal{O}^\gamma$, $0 \leq \gamma \leq k$, is a bounded domain with C^∞-boundary $\partial\mathcal{O}^\gamma$. We also assume that $\mathscr{H}_2(\mathcal{O}^\gamma) = \{0\}$, $1 \leq \gamma \leq k$. Take open subsets $\{\mathcal{O}_i^\gamma\}_{\gamma=0}^k$, $i = 1,2$, such that $\cup_{\gamma=0}^k \mathcal{O}_1^\gamma = \Omega$ and

$$\mathcal{O}_1^0 \subset\subset \mathcal{O}_2^0 \subset\subset \mathcal{O}^0,\ \overline{\mathcal{O}_1^\gamma} \subset\subset \overline{\mathcal{O}_2^\gamma} \subset\subset \overline{\mathcal{O}^\gamma} \text{ in } \bar{\Omega},\ 1 \leq \gamma \leq k.$$

Choose cut off functions $\{\varphi_i^\gamma\}_{\gamma=0}^k$, $i = 1,2$, satisfying supp $\varphi_i^0 \subset \mathcal{O}_i^0$, and supp $\varphi_i^\gamma \subset \mathcal{O}_i^\gamma \cup \Gamma$, $1 \leq \gamma \leq k$, for $i = 1, 2$. In addition, these $\{\varphi_i^\gamma\}_{\gamma=0}^k$, $i = 1,2$, are chosen so as to satisfy $\sum_{\gamma=0}^k \varphi_1^\gamma = 1$ on Ω, $\varphi_2^\gamma = 1$ on $\overline{\mathcal{O}_1^\gamma}$ for $0 \leq \gamma \leq k$, and $\partial\varphi_2^\gamma/\partial n = 0$ on Γ for $1 \leq \gamma \leq k$. Next we solve the following boundary value problems:

$$(7)\quad \begin{aligned} &\Delta_2 v^0 = f^0 \text{ in } \mathcal{O}^0,\ v^0 = 0 \text{ on } \partial\mathcal{O}^0, \\ &\Delta_2 v^\gamma = f^\gamma \text{ in } \mathcal{O}^\gamma,\ \iota_\gamma^*(*v^\gamma) = \iota_\gamma^*(*d_2 v^\gamma) = 0,\ 1 \leq \gamma \leq k, \end{aligned}$$

where the f^γ are assumed to be in $H^0(\mathcal{O}^\gamma; \Lambda^2)$, $0 \leq \gamma \leq k$, and the ι_γ denote the inclusion maps $\partial\mathcal{O}^\gamma \to \overline{\mathcal{O}^\gamma}$, $1 \leq \gamma \leq k$. Since $\mathscr{H}_2(\mathcal{O}^\gamma) = \{0\}$, $1 \leq \gamma \leq k$, we see from ii) in §2 that the problems (7) have solutions $v^\gamma \in H^2(\mathcal{O}^\gamma; \Lambda^2)$, $0 \leq \gamma \leq k$. Using conventional notations, we may rewrite (7) as follows:

$$(8)\quad \begin{aligned} &-\Delta v^0 = f^0 \text{ in } \mathcal{O}^0,\ v^0 = 0 \text{ on } \partial\mathcal{O}^0, \\ &-\Delta v^\gamma = f^\gamma \text{ in } \mathcal{O}^\gamma,\ v^\gamma \times n^\gamma = 0,\ \nabla \cdot v^\gamma = 0 \text{ on } \partial\mathcal{O}^\gamma,\ 1 \leq \gamma \leq k. \end{aligned}$$

Here $\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2$ and $n^\gamma = n^\gamma(x) = (n_1^\gamma, n_2^\gamma, n_3^\gamma)$ is the unit outward normal at $x \in \partial\mathcal{O}^\gamma$. Notice that $-\Delta$ is an elliptic operator and the boundary conditions in (8) satisfy the complementing condition with respect to $-\Delta$ in the sense of [1]. Then by virtue of Theorem 5.1 of [8], we see that there exist 3×3 matrices $g^\gamma(x, y)$ defined on $\overline{\mathcal{O}^\gamma} \times \overline{\mathcal{O}^\gamma}$, $0 \leq \gamma \leq k$, such that the solutions v^γ of (8) are expressed as

$$(9)\quad v^\gamma(x) = \int_{\mathcal{O}^\gamma} g^\gamma(x, y) f^\gamma(y)\,dy,\ 0 \leq \gamma \leq k,$$

and the following estimates hold for any multi-indices α, β:

$$(10)\quad |\partial_x^\alpha \partial_y^\beta g^\gamma(x, y)| \leq C|x - y|^{-1-|\alpha|-|\beta|},\ (x, y) \in \overline{\mathcal{O}^\gamma} \times \overline{\mathcal{O}^\gamma},\ 0 \leq \gamma \leq k.$$

Fix $t \in [0, T]$. Let each $\varphi_2^\gamma(x) g^\gamma(x, y)$, $0 \leq \gamma \leq k$, be the extension with respect to x of itself taking the value 0 outside of $\overline{\mathcal{O}^\gamma}$. We put $q(x, y) = \sum_{\gamma=0}^k \varphi_2^\gamma(x) g^\gamma(x, y) \varphi_1^\gamma(y)$ and then set

$$(11)\quad R[d_1 u](t, x) = \Delta \int_\Omega q(x, y) d_1 u(t, y)\,dy + d_1 u(t, x).$$

Here $\int_\Omega q(x, y) d_1 u(t, y)\,dy \in D(\Delta_2)$, as a 2-form, by a particular choice of φ_2^γ, $1 \leq \gamma \leq k$. Referring to (8), (9), we get by direct calculation

$$(12)\quad R[d_1 u](t, x) = \int_\Omega r(x, y) d_1 u(t, y)\,dy.$$

Here $r(x, y)$ is a 3×3 matrix depending on $x, y \in \Omega$, which consists of first order derivatives of $g^\gamma(x, y)$ with respect to x multiplied by $\varphi_1^\gamma(y)$ and the derivatives of $\varphi_2^\gamma(x)$, $0 \leq \gamma \leq k$. So, thanks again to the special choice of $\{\varphi_i^\gamma\}_{\gamma=0}^k$, $i = 1, 2$, we see that $r(x, y)$ is smooth on $\Omega \times \Omega$. On the other hand, since both terms on the right side of (11) belong to $\mathscr{H}_2(\Omega)^{\perp}$, $R[d_1u] \in \mathscr{H}_2(\Omega)^{\perp}$. Hence, in view of ii) in §2 and the fact remarked after (11), we finally obtain from (11)

$$(13)\quad \delta_2\Delta_2^{-1}(d_1u)(t, x) = \delta_2 \int_{\Omega} q(x, y) d_1u(t, y) dy + \delta_2\Delta_2^{-1}(R[d_1u])(t, x).$$

4. We give a sketch of the proof of (4). Fix $t \in [0, T]$. Note that the pressure $p(t, x)$ is a solution of a certain Neumann problem in Ω (see the proof of Theorem 3 in [3]). Then by using Gagliardo-Nirenberg's inequality and by applying a limit argument, we obtain (see [2] for the counterpart of this inequality)

$$(14)\quad \| u(t) \|_s \leq \| u(0) \|_s \exp(C \int_0^t \{ | \nabla u(\tau) |_{L^\infty(\Omega)} + | u(\tau) |_{L^\infty(\Omega)} \} d\tau).$$

This estimate is given in [10]. In addition, we have

$$(15)\quad | \nabla u(t) |_{L^\infty(\Omega)} \leq C \{ \| u(0) \|_0 + 1 + (1 + \log_+ \| u(t) \|_3) | d_1u(t) |_{L^\infty(\Omega)} \},$$

$$(16)\quad | u(t) |_{L^\infty(\Omega)} \leq C \{ \| u(0) \|_0 + | d_1u(t) |_{L^\infty(\Omega)} \}.$$

Here $\log_+ r := \log r$ for $r \geq 1$, $:= 0$ for $0 \leq r < 1$. Noting that $| d_1u(t) |_{L^\infty(\Omega)} = | \mathrm{rot}\, u(t) |_{L^\infty(\Omega)}$, and combining (14)-(16), we get the desired estimate (4) in the same way as in [2]. So we give the proof of the estimate (15). The estimate (16) is proved more directly. First, since the terms of (6) are L^2-orthogonal, we obtain from the fact that $\| u(t) \|_0 = \| u(0) \|_0$

$$(17)\quad | \nabla \sum_{i=1}^{R} \lambda_i(t) a_i(x) |_{L^\infty(\Omega)} \leq \| u(0) \|_0 \sum_{i=1}^{R} | \nabla a_i(x) |_{L^\infty(\Omega)}.$$

Next, since $r(x, y)$ in (12) is smooth on $\Omega \times \Omega$, we get by using Sobolev's inequality and Theorem 10.5 in [1]

$$(18)\quad \begin{aligned} | \nabla\delta_2\Delta_2^{-1}(R[d_1u])(t) |_{L^\infty(\Omega)} &\leq C | \nabla\delta_2\Delta_2^{-1}(R[d_1u])(t) |_{W^{1,p}(\Omega)} \\ &\leq C | R[d_1u](t) |_{W^{1,p}(\Omega)} \leq C | d_1u(t) |_{L^\infty(\Omega)}, \end{aligned}$$

for $p > 3$. In view of (6), (13), it remains to show pointwise estimates of the gradient of $\delta_2\varphi_2^{\gamma}(x) w^{\gamma}(t, x)$, $0 \leq \gamma \leq k$, where $w^{\gamma}(t, x) = \int_{\mathcal{O}^{\gamma}} g^{\gamma}(x, y) \varphi_1^{\gamma}(y) \times d_1u(t, y) dy$. To do this, we introduce diffeomorphisms $\{\Phi^{\gamma}\}_{\gamma=1}^{k}$ such that each Φ^{γ} maps $\mathcal{O}^{\gamma}$ onto V^{γ} which is contained in $\mathscr{B}_+ = \{x \mid |x| < 1, x^3 > 0\}$, and $\overline{\mathcal{O}^{\gamma}} \cap \Gamma$ corresponds to a part of $\sigma = \{x \mid |x| < 1, x^3 = 0\}$. In addition, $\{(\mathcal{O}^{\gamma}, \Phi^{\gamma})\}_{\gamma=1}^{k}$ must be taken so as to be an admissible boundary coordinate system (see Definition 7.5.2 and Lemma 7.5.1 in [7]). Fix $1 \leq \gamma \leq k$ and omit suffix γ from $\mathcal{O}^{\gamma}$, w^{γ}, Φ^{γ}, and so on. It is easy to see that w satisfies the boundary conditions of (8) on $\overline{\mathcal{O}} \cap \Gamma$, if and only if $\tilde{w}_1 = \tilde{w}_2 = \partial\tilde{w}_3/\partial\tilde{x}^3 = 0$ on $\bar{V} \cap \sigma$ where $\tilde{x} = \Phi(x)$ and $\tilde{w}(t, \tilde{x}) = (\Phi^{-1})^* w(t, x)$ as 2-forms. Then we observe that each of $\partial\tilde{w}/\partial\tilde{x}^i$, $i = 1,2$, satisfies also the same boundary conditions as above on $\bar{V} \cap \sigma$. Define two differential operators $\Lambda_i = \sum_{j=1}^{3} b_i^j(x) \partial_j E + B_i(x)$, $i = 1,2$, which act on 2-forms v on $\mathcal{O}$, by $\Lambda_i v = \Phi^*(\partial\tilde{v}/\partial\tilde{x}^i)$. Here $b_i^j(x)$, E, $B_i(x)$ are smooth functions on $\mathcal{O}$, the 3×3 unit matrix, 3×3 matrices depending smoothly on $x \in \mathcal{O}$, respectively. It is obvious that $\sum_{j=1}^{3} b_i^j n_j = 0$ on $\bar{\mathcal{O}} \cap \Gamma$, $i = 1,2$, and each $\Lambda_i w$ satisfies the same boundary conditions as in (8) on $\bar{\mathcal{O}} \cap \Gamma$. Hence we have

$$(19)\quad \begin{aligned} &-\Delta(\Lambda_i\varphi_2 w) = F_i(t, x) \text{ in } \mathcal{O}, \\ &(\Lambda_i\varphi_2 w)\times n = 0,\ \nabla\cdot(\Lambda_i\varphi_2 w) = 0 \text{ on } \partial\mathcal{O},\ i = 1,2, \end{aligned}$$

where $F_i(t, x) = (-\Lambda_i\varphi_1 d_1 u + \Lambda_i[-\Delta, \varphi_2]w + [-\Delta, \Lambda_i]\varphi_2 w)(t, x)$. Then, by the same reasoning as used to show (9), we obtain $(\Lambda_i\varphi_2 w)(t, x) = \int_{\mathcal{O}} g(x, y)F_i(t, y)dy$, $i = 1,2$. So, regarding δ_2 as a system of differential operators (see (7.2.14) in [7]), we have for $i = 1,2$,

$$(20)\quad (\Lambda_i\delta_2\varphi_2 w)(t, x) = \int_{\mathcal{O}} \delta_{2,x}\, g(x, y)F_i(t, y)dy - [\delta_2, \Lambda_i]\varphi_2 w(t, x).$$

Note that Λ_i is also defined on 1-forms by the same way as above. To estimate the first term on the right side of (20), we introduce an auxiliary function $\zeta(x) \in C_0^\infty(\boldsymbol{R}^3)$ such that $\zeta := 1$ for $|x| \leq 1$, $:= 0$ for $|x| \geq 2$, and divide this term into two terms,

$$-\int_{\mathcal{O}} \zeta((x-y)/\rho)\delta_{2,x}\, g(x, y)(\Lambda_i\varphi_i d_1 u)(t, y)dy$$

$$-\int_{\mathcal{O}} \{1-\zeta((x, y)/\rho)\}\delta_{2,x}\, g(x, y)(\Lambda_i\varphi_1 d_1 u)(t, y)dy,$$

where $0 < \rho \leq \rho_0$ with ρ_0 small enough. Taking $\rho := \rho_0$ if $\| u \|_3 \leq \rho_0^{-1/4}$, $:= \| u \|_3^{-4}$ otherwise, as in pp. 65-66 in [2] we obtain from (10)

$$(21)\quad \begin{aligned} &\left| \int_{\mathcal{O}} \delta_{2,x}\, g(x, y)(\Lambda_i\varphi_1 d_1 u)(t,y)dy \right|_{L^\infty(\mathcal{O})} \\ &\leq C\{1 + \log_+ \| u(t) \|_3\} \,|\, d_1 u(t, x) \,|_{L^\infty(\Omega)} + C. \end{aligned}$$

By arguments similar to that used for deriving (18), we get

$$(22)\quad \begin{aligned} &|\, [\delta_2, \Lambda_i]\varphi_2 w(t) \,|_{L^\infty(\Omega)} \leq C\,|\, [\delta_2, \Lambda_i]\varphi_2 w(t) \,|_{W^{1,p}(\Omega)} \\ &\leq C\,|\, w(t) \,|_{W^{2,p}(\Omega)} \leq C\,|\, d_1 u(t) \,|_{L^p(\Omega)} \leq C\,|\, d_1 u(t) \,|_{L^\infty(\Omega)},\ i = 1, 2, \end{aligned}$$

for $p > 3$. Further, by Hölder's inequality and Theorem 10.5 in [1], we deduce from (10) that

$$(23)\quad \begin{aligned} &\left| \int_{\mathcal{O}} \delta_{2,x}\, g(x, y)(\Lambda_i[-\Delta, \varphi_2]w + [-\Delta, \Lambda_i]\varphi_2 w)(t,y)dy \right|_{L^\infty(\mathcal{O})} \\ &\leq C\,|\, w(t) \,|_{W^{2,4}(\Omega)} \leq C\,|\, d_1 u(t) \,|_{L^4(\Omega)} \leq C\,|\, d_1 u(t) \,|_{L^\infty(\Omega)},\ i = 1,2. \end{aligned}$$

Accordingly, we conclude from (6), (13), (17)-(18), and (21)-(23) that the maximum norms of two tangential derivatives on $\mathcal{O}$ are estimated by the terms of the right side of (15). Since the normal derivative of a solenoidal vector field is expressed as a sum of the tangential derivatives and the components of the vorticity, we get finally the estimate (15) on a suitable neighborhood of the boundary. We can also prove the estimate (15) on a subset of Ω far from the boundary in a similar way. Thus we end the proof of (15).

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