16. *A Table of the Dimensions of the Extended Hilbert Modular Type Cusp Forms*

By Hirofumi ISHIKAWA
Department of Mathematics, College of Arts and Sciences, Okayama University

(Communicated by Shokichi IYANAGA, M. J. A., Feb. 13, 1990)

1. Introduction and the table. For a square-free positive number D, let k be a real quadratic number field $\boldsymbol{Q}(\sqrt{D})$. Let $\mathfrak{o}$, U and U^+ be the ring of integers in k, the group of units in $\mathfrak{o}$ and the group of all totally positive units. The extended Hilbert modular group is defined as follows

(1) $$\hat{\Gamma}=\{\gamma\in GL_2(\mathfrak{o})\,;\,\det(\gamma)\in U^+\}\Big/\left\{\begin{bmatrix}\varepsilon, & 0\\ 0, & \varepsilon\end{bmatrix};\ \varepsilon\in U\right\}.$$

Hausmann investigated the fixed points of $\hat{\Gamma}$ in [1]. When k has a unit of negative norm, $\hat{\Gamma}$ coincides with the ordinary Hilbert modular group Γ. We consider the space $\hat{S}(D)$ of the cusp forms of weight two with respect to $\hat{\Gamma}$ in H^2 (H being a complex upper half plane).

For the ordinary Hilbert modular group, we have already given a dimension table in [5] of which this note is a continuation. We tabulate the dimension of $\hat{S}(D)$ for a square-free D and $1<D<1000$. In the following table, the number D is given by

(2) $$D=i+j \qquad (i=\text{row number},\ j=\text{column number}).$$

When the mark '$-$' appears after a figure, $\boldsymbol{Q}(\sqrt{D})$ has a unit of negative norm. The mark '$**$' means that D is not square-free. To calculate this table, we used ACOS-6 computer system in Okayama University Computer center.

2. The method of the computation. From now on, we will only treat with the case of $\hat{\Gamma}\neq\Gamma$. For a square-free divisor w of the discriminant d_k of k, let Γ_w be the subgroup of $PL_2(k)$ generated by Γ and the set of elements $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ (mod k^X) such that $a,\ b,\ c,\ d\in(w)^{1/2}$, $ad-bc=w$, where $(w)^{1/2}$ is an ideal whose square equals (w). When $\tilde{w}$ is a square-free part of d_k/w, $\Gamma_w=\Gamma_{\tilde{w}}$. There exists some w such that $\Gamma_w=\hat{\Gamma}$.

By virtue of [1], [3], we get

Theorem. *Let w be a divisor of d_k satisfying $\Gamma_w=\hat{\Gamma}$. The dimension of $\hat{S}(D)$ is given by*

(3) $$\dim\hat{S}(D)=t_0+t_1+t_2-1$$

Each term can be written as follows.

(4) $$t_0=(1/4)\zeta_k(-1)$$

(5) $$t_1=a(D,w)h(-D)+b(D,w)h(-3D)+c(D,w)h(-w)h(-\tilde{w})$$

$$(6)\qquad t_2=\begin{cases}0 & \text{if } D \text{ has no primes} \equiv 3 \bmod 4\\ -2\sum h(-d_1)h(-d_2)/u(-d_1)u(-d_2) & \text{otherwise}\end{cases}$$

where ζ_k *is the Dedekind zeta function of* k, (d_1, d_2) *runs over all discriminants of imaginary quadratic fields satisfying* $d_k=d_1\cdot d_2$, *and* $h(-d)$, $u(-d)$ *denote a class number of* $\boldsymbol{Q}(\sqrt{-d})$, *an order of the unit group of* $\boldsymbol{Q}(\sqrt{-d})$. $a(D,w)$, $b(D,w)$ *and* $c(D,w)$ *are given in the following tables.*

D w (or $\tilde{w}$)	$D\equiv1(4)$	$D\equiv2(4)$ $w\neq2$	 $w=2$	$D\equiv3(8)$, $D\neq3$ $w\neq2$	 $w=2$	$D\equiv7(8)$	$D=3$ $w=2$
$16a(D,w)$	1	3	5	10	22	4	21

D w (or $\tilde{w}$)	$D\equiv1,2(3)$	$D\equiv3(9)$, $D\neq3$ $w\neq3$	 $w=3$	$D\equiv6(9)$ $w\neq3$	 $w=3$	$D=3$ $w=3$
$48b(D,w)$	4	16	34	8	2	17

D w (or $\tilde{w}$)	$D\equiv1(8)$ $w\equiv1(4)$	 $w\equiv3(8)$ $w\neq3$	 $w\equiv7(8)$	 $w=3$	$D\equiv5(8)$ $w\equiv1(4)$	 $w\equiv3(4)$ $w\neq3$	 $w=3$
$8c(D,w)$	1	16	4	5	1	4	1

$D\equiv2(4)$ $w\equiv1(4)$	 $w\equiv3(8)$ $w\neq3$	 $w\equiv7(8)$	 $w=3$	$D\equiv3(4)$, $D\neq3$ $w\equiv3(8)$ $w\neq3$	 $w\equiv7(8)$	 $w\equiv0(2)$	 $w=3$	$D=3$ $w=2$
3	10	4	3	10	4	1	3	1

Remarks. i) w. If an ideal in k whose norm equals w ($w\neq1$, or D) becomes principal, Γ_w coincides with $\hat{\Gamma}$. Then we have to seek such a 'w' through $w \mid d_k$.

ii) t_0, t_2. The method of their calculations was given in [5].

iii) t_1. There are elliptic points of order 2, 3, 4, 6, 12. A point of order 4, 6, or 12 appears only when w or $\tilde{w}=2$, w or $\tilde{w}=3$, or $D=3$, respectively. These contributions are expressed by class numbers of imaginary quadratic fields.

Table

	0	100	200	300	400	500	600	700	800	900
1	**	4–	5	6	24–	10	50–	39–	**	59–
2	0–	11	55–	45	71	110	112	**	226	206
3	0	12	23	44	81	85	**	175	180	220
5	0–	1	6	9	**	22	**	24	25	39
6	0	25–	29	**	83	97	142	196	188	242
7	0	13	**	56	65	**	140	138	196	269

Table (continued)

9	**	4–	6	8	28–	23–	22	43–	67–	**
10	1–	10	23	53	75	91	304–	145	**	250
11	1	11	36	45	80	119	127	**	233	227
13	0–	4–	2	18–	9	**	35–	24	26	47
14	1	13	36	96–	**	124	134	163	238	470–
15	0	10	25	**	83	94	119	174	183	219
17	0–	**	5	13–	15	13	41–	15	37	24
18	**	16	56–	46	83	89	126	188	358–	**
19	2	9	34	55	80	108	161	155	**	288
21	0	**	5	10	21–	35–	**	32	46–	45
22	2	24–	26	55	72	**	142	**	198	520–
23	1	14	34	47	**	119	111	167	232	201
26	4–	**	73–	56	80	126	254–	**	233	242
27	**	16	32	51	85	87	134	188	187	**
29	1–	3	9–	9	6	**	31–	**	52–	78–
30	2	32–	29	48	86	196–	**	391–	190	222
31	2	18	28	67	73	**	166	171	211	**
33	0	2	11–	**	29–	25–	25	42–	**	24
34	4	18	**	64	71	118	332–	164	224	288
35	2	**	34	49	72	113	120	**	230	221
37	1–	5–	6	21–	9	19	**	28	**	90–
38	4	14	35	**	80	228–	127	**	226	214
39	3	21	31	63	92	**	**	212	191	271
41	1–	1	13–	7	**	30–	48–	20	**	53–
42	2	18	**	**	181–	100	139	187	385–	235
43	5	13	**	**	81	115	158	153	202	257
45	**	6–	**	8	23–	19	16	36	**	**
46	4	20	40	130–	84	111	175	345–	**	300
47	3	**	41	57	74	128	125	**	**	230
49	**	6–	7	16–	28–	**	32	19	41	65–
51	5	20	41	**	100	99	147	214	208	258
53	2–	**	5	18–	12	20	33–	32	51–	79–
54	**	18	38	63	104	221–	147	408–	210	**
55	5	17	29	65	69	110	171	159	**	279
57	1	6–	12–	6	32–	27–	**	47–	65–	25
58	10–	18	38	70	153–	**	147	168	200	275
59	7	18	43	57	**	131	152	178	263	235
61	2–	3	**	**	21–	21	37–	59–	26	**
62	6	**	44	107–	73	139	140	168	245	444–
63	**	24	35	**	100	110	131	199	195	**
65	2–	2	14–	17–	14	29–	22	**	84–	55–
66	7	26	36	63	109	116	**	215	215	267

Table (continued)

67	8	17	40	70	86	**	165	163	**	288
69	0	**	11-	**	12	41-	20	71-	24	47
70	7	41-	**	138-	83	108	164	160	224	593-
71	5	**	46	59	97	147	135	203	259	259
73	2-	7-	6	19-	15	11	57-	38-	**	31
74	15-	23	98-	65	92	139	153	**	256	259
77	1	4	13-	11	**	43-	34-	30	56-	80-
78	5	29	40	**	103	**	149	411-	195	257
79	8	26	**	81	83	125	177	188	234	311
81	**	7-	15-	10	36-	14	29	25	74-	**
82	16-	20	41	73	89	119	159	159	**	300
83	10	25	51	57	77	137	143	**	262	225
85	3-	8-	4	13	23-	**	39-	60-	24	100-
86	10	24	43	71	**	289-	**	204	274	495-
87	6	27	35	**	106	115	157	208	193	249
89	3-	**	**	18-	17	17	27	20	44	30
90	**	24	84-	59	**	116	150	211	219	**
91	11	23	51	73	101	124	188	175	**	322
93	2	9-	12-	13	25-	38-	**	37	25	45
94	11	29	**	165-	93	**	192	364-	242	302
95	9	20	55	61	**	129	147	188	257	251
97	3-	8-	**	18-	15	18	59-	41-	35	65-
98	**	**	103-	63	98	131	282-	175	266	248
99	**	30	43	70	120	123	172	221	207	**

References

[1] Hausmann, W.: Kurven auf Hilbertschen Modulflächen. Bonner Math. Schr., **123** (1980).

[2] ——: The fixed point of the symmetric Hilbert modular group of a real quadratic field with arbitrary discriminant. Math. Ann., **260**, 31–50 (1982).

[3] Hirzebruch, F. E. P.: Hilbert modular surfaces. L'Enseignement Math., **19**, 183–281 (1973); Gesammelte Abhandlungen Bd., **2**, 225–323 (1987).

[4] Ishikawa, H.: The traces of Hecke operators in the space of the 'Hilbert Modular' type cusp forms of weight two. Scientific Papers C. General Education Univ. Tokyo, **29**, 1–28 (1979).

[5] ——: A table of the dimensions of the Hilbert modular type cusp forms over real quadratic fields. Proc. Japan Acad., **64**A, 84–87 (1988).

[6] Prestel, A.: Die elliptischen Fixpunkte der Hilbertschen Modulgruppen. Math. Ann., **177**, 181–209 (1968).

[7] ——: Die Fixpunkte der symmetrischen Hilbertschen Modulgruppe zu einem reell quadratischen Zahlkörper mit Primzahldiskriminante. ibid., **200**, 123–139 (1973).

[8] Shimizu, H.: On discontinuous groups operating on the product of the upper half planes. Ann. of Math., **77**, 33–71 (1963).