68. *Support of CR-hyperfunctions*

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In this note we examine the possible shape of support of **CR**-hyperfunctions.

Let X be a complex m-dimensional complex manifold, and let N be a real k-codimensional real analytic submanifold of X, with $0 \leqq k \leqq m$. Throughout this note we assume that the submanifold N is a *generic* **CR**-submanifold (see [2]).

Let $\bar{\partial}_b$ be the tangential Cauchy-Riemann system induced on N. A hyperfunction h on N which satisfies the equations $\bar{\partial}_b h = 0$ is called a **CR**-hyperfunction. We denote by supp (h) the support of **CR**-hyperfunction h.

Remark 1. Every **CR**-hyperfunction defined on a Levi-flat **CR**-submanifold is a hyperfunction with holomorphic parameters (cf. [4]).

Let Y be a complexification of N and let $T_N^*Y\,(=\sqrt{-1}T^*N)$ be the conormal bundle of N in the cotangent vector bundle T^*Y of Y. Let us denote by $SS(\bar{\partial}_b)$ the characteristic variety of the tangential Cauchy-Riemann system $\bar{\partial}_b$.

Note that the purely imaginary locus of the characteristic variety, denoted by $SS(\bar{\partial}_b) \cap T_N^*Y$, is a real $2m$ dimensional manifold (cf. Proposition 1.2.1 of [6]).

Let L be a real analytic submanifold of N and let Z be its complexification. Then we have the following exact sequences (cf. [5]):

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & & & \\
 & & \downarrow & & \downarrow & & & & \\
0 & \longrightarrow & \sqrt{-1}T_L^*N & \longrightarrow & T_Z^*Y|_L & \longrightarrow & T_L^*N & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \| & & \\
0 & \longrightarrow & T_N^*Y|_L & \longrightarrow & T_L^*Y & \longrightarrow & T_L^*N & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & & & \\
 & & T_L^*Z & = & T_L^*Z & & & & \\
 & & \downarrow & & \downarrow & & & & \\
 & & 0 & & 0 & . & & &
\end{array}
$$

Here we identify $T_N^*Y|_L \cap T_Z^*Y|_L$ with $\sqrt{-1}T_L^*N$.

Definition 2. A real analytic submanifold L of N is said to be totally characteristic, if the purely imaginary conormal bundle $\sqrt{-1}T_L^*N$ satisfies the following condition:

$$\sqrt{-1}T_L^*N \subseteq SS(\bar{\partial}_b) \cap T_N^*Y.$$

Lemma 3. *If L is a totally characteristic submanifold of N, then the inequality* $\dim_{\mathbf{R}} L \geqq 2m-2k$ *holds.*

Proof. Let $p \in L$. If we denote by $\sqrt{-1}T_L^*N_p$ (resp. by $(SS(\bar{\partial}_b) \cap T_N^*Y)_p$) the fiber over p of the bundle $\sqrt{-1}T_L^*N$ (resp. $SS(\bar{\partial}_b) \cap T_N^*Y$), we have $\dim_{\mathbf{R}} \sqrt{-1}T_L^*N_p = \dim_{\mathbf{R}} N - \dim_{\mathbf{R}} L$ and $\dim_{\mathbf{R}} (SS(\bar{\partial}_b) \cap T_N^*Y)_p = k$. Hence we have the inequality $2m-k-\dim_{\mathbf{R}} L \leqq k$ which yields the conclusion. Q.E.D.

The main result of this note is the following theorem.

Theorem 4. *Let h be a non-zero* **CR**-*hyperfunction on the generic* **CR**-*submanifold N. Assume that* supp(h), *denoted by L, is a real analytic submanifold of N. Then L is a totally characteristic submanifold of N.*

Sketch of the proof. Let us denote by Z a complexification of L. An interdependence of the support and the singular spectrum of h (see [5], Proposition 3.5.2 of [3]) implies

$$\sqrt{-1}S_L^*N \subseteq SS(h),$$

where $SS(h)$ denotes the singular spectrum of h. Sato's fundamental theorem implies

$$SS(h) \subseteq SS(\bar{\partial}_b) \cap S_N^*Y.$$

Hence we have

$$\sqrt{-1}S_L^*N \subseteq SS(\bar{\partial}_b) \cap S_N^*Y. \qquad \text{Q.E.D.}$$

Corollary 5. *Under the assumptions of Theorem* 4, *we have*

$$\dim_{\mathbf{R}} \operatorname{supp}(h) \geqq 2m-2k.$$

By the same argument as the proof of Theorem 17.1 of [1], we can conclude the following result.

Corollary 6. *Let h be a* **CR**-*hyperfunction on the generic* **CR**-*submanifold N. Suppose that* supp(h) *is a $2m-2k$ dimensional real analytic submanifold of N. Then* supp(h) *is a complex $m-k$ dimensional submanifold.*

References

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