101. On Algebraic K3 Surfaces with Finite Automorphism Groups

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1. Introduction. By a *surface* we shall always mean a nonsingular compact complex surface. An algebraic surface X is called a $K3$ *surface* if $\dim H^1(X, \mathcal{O}_X)=0$ and the canonical line bundle K_X is trivial. It is known that the automorphism group $\mathrm{Aut}(X)$ of X is isomorphic, up to a finite group, to the factor group $O(S_X)/W(S_X)$, where $O(S_X)$ is the automorphism group of the Picard lattice S_X of X (i.e. S_X is the Picard group of X together with the intersection form) and $W(S_X)$ is its subgroup generated by all reflections associated with elements with square (-2) of S_X ([6]). Recently Nikulin [4], [5] has completely classified the Picard lattices of algebraic $K3$ surfaces with finite automorphism groups.

Our goal is to compute the automorphism groups of algebraic $K3$ surfaces with finite automorphism groups. The proof will be given elsewhere.

2. Main result. A *lattice* L is a free $\boldsymbol{Z}$-module of finite rank endowed with an integral bilinear form. By $L_1 \oplus L_2$ we denote the orthogonal direct sum of lattices L_1 and L_2. For a lattice L and an integer m we denote by $L(m)$ the lattice whose bilinear form is the one on L multiplied by m. Also we denote by U the lattice of rank 2 with the intersection matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and by A_m, D_n and E_k the negative definite lattices associated with the Dynkin diagram of type A_m, D_n and E_k respectively.

Let X be an algebraic $K3$ surface. The second cohomology group $H^2(X, \boldsymbol{Z})$ admits a canonical structure of a lattice induced from the cup product. The Picard lattice S_X of X is then naturally embedded in the lattice $H^2(X, \boldsymbol{Z})$. We denote by T_X the orthogonal complement of S_X in $H^2(X, \boldsymbol{Z})$, which is called a *transcendental lattice* of X.

In the following we assume that the automorphism group $\mathrm{Aut}(X)$ of X is finite. By definition, there exists a nowhere vanishing holomorphic 2-form ω_X on X. An automorphism g of X acts on $\boldsymbol{C}\{\omega_X\}$ as $g^*\omega_X=\alpha_X(g)\cdot\omega_X$ where $\alpha_X(g) \in \boldsymbol{C}^*$. Therefore we have an exact sequence

$$(1) \qquad 1 \longrightarrow G_X \longrightarrow \mathrm{Aut}(X) \xrightarrow{\alpha_X} \boldsymbol{Z}/m \longrightarrow 1$$

where $\boldsymbol{Z}/m$ is a cyclic group of m-th root of unity in $\boldsymbol{C}^*$ and G_X is the kernel of α_X. Moreover the representation of the cyclic group $\boldsymbol{Z}/m$ in $T_X \otimes \boldsymbol{Q}$ is isomorphic to a direct sum of irreducible representations of the cyclic group $\boldsymbol{Z}/m$ over $\boldsymbol{Q}$ of maximal rank $\phi(m)$, where ϕ is the Euler function. In particular $\phi(m) \leqq \mathrm{rank}(T_X)$ and hence $m \leqq 66$ ([3], Theorem 3.1).

Definition. An algebraic $K3$ surface X is called *general* if the image of α_X is at most of order 2, and X is called *special* if it is not general. The meaning of this definition is as follows: Let X be an algebraic $K3$ surface with a Picard lattice S_X. Let S be an abstract lattice which is isomorphic to S_X. Denote by M_S the moduli space for algebraic $K3$ surfaces whose Picard lattices are isomorphic to S. Then the dimension of M_S is equal to $20-\mathrm{rank}(S)$. A general $K3$ surface Y with $S_Y=S$ corresponds to a point of the complement of hypersurfaces in M_S.

Theorem. *Let X be an algebraic $K3$ surface with finite automorphism group* $\mathrm{Aut}(X)$. *Then*

(i) *If X is general, then* $\mathrm{Aut}(X)$ *is as in the following table*:

S_X			$\mathrm{Aut}(X)$
$U\oplus E_8\oplus E_8\oplus A_1$			$\mathfrak{S}_3\times Z/2$
$U\oplus E_8\oplus E_8$,	$U\oplus E_8\oplus E_7$,	$U\oplus E_8\oplus D_6$,	$Z/2\times Z/2$,
$U\oplus E_8\oplus D_4\oplus A_1$,	$U\oplus D_8\oplus D_4$,	$U\oplus E_8\oplus A_1^4$,	
$U\oplus E_7\oplus A_1^4$,	$U\oplus D_6\oplus A_1^4$,	$U\oplus D_4\oplus A_1^6$,	
$U\oplus D_4\oplus A_1^5$,	$U(2)\oplus D_4\oplus D_4$,	$U\oplus A_1^8$,	
$U(2)\oplus A_1^7$			
otherwise			$Z/2$ *or* $\{1\}$

where A_1^k denotes the direct sum $A_1\oplus\cdots\oplus A_1$ (k times).

(ii) *If X is special, then* $\mathrm{Aut}(X)$ *is a cyclic extension of the group in the above table. This extension is given by the exact sequence* (1).

The main tools used in the proof are Nikulin's analysis on finite automorphisms of $K3$ surfaces [3] and the theory of elliptic surfaces due to Kodaira [1] and Shioda [7].

Remark. There exists a special algebraic $K3$ surface whose automorphism group is isomorphic to $Z/66$. This automorphism acts on the Picard group as identity. In [2], we shall study an automorphisms with this property.

References

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