97. *Spectral Properties of Random Media*

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This note is a continuation of our previous papers Ozawa [2], [3]. We consider a bounded domain Ω in $\boldsymbol{R}^3$ with smooth boundary Γ. We put $B(\varepsilon; w)=\{x\in \boldsymbol{R}^3; |x-w|<\varepsilon\}$. Fix $\beta\geq 1$. Let $0<\mu_1(\varepsilon; w(m))\leq\mu_2(\varepsilon; w(m))\leq\cdots$ be the eigenvalues of $-\Delta$ $(=-\mathrm{div\ grad})$ in $\Omega_{\varepsilon, w(m)}=\Omega\backslash\overline{\bigcup_{i=1}^{\tilde{m}} B(\varepsilon; w_i^{(m)})}$ under the Dirichlet condition on its boundary. Here $\tilde{m}$ denotes the largest integer which does not exceed m^β, and $w(m)$ denotes the set of $\tilde{m}$-points $\{w_i^{(m)}\}_{i=1}^{\tilde{m}}\in\Omega^{\tilde{m}}$. Let $V(x)>0$ be C^1-class function on $\bar{\Omega}$ satisfying

$$\int_\Omega V(x)dx=1.$$

We consider Ω as the probability space with the probability density $V(x)dx$. Let $\Omega^{\tilde{m}}=\prod_{i=1}^{\tilde{m}}\Omega$ be the probability space with the product measure.

In this note we give the following:

Theorem 1. *Fix $\beta\in[1,3)$. Assume that $V(x)>0$. Fix $\alpha>0$ and k. Then, there exists a constant $\delta(\beta)>0$ independent of m such that*

$$\lim_{m\to\infty} \boldsymbol{P}(w(m)\in\Omega^{\tilde{m}}; m^{\delta'-(\beta-1)}|\mu_k(\alpha/m; w(m))-\mu_{k,m}^V|<\varepsilon)=1$$

holds for any $\varepsilon>0$ and $\delta'\in[0,\delta(\beta))$. Here $\mu_{k,m}^V$ denotes the k-th eigenvalue of $-\Delta+4\pi\alpha m^{\beta-1}V(x)$ in Ω under the Dirichlet condition on Γ.

We give a short sketch of our proof of Theorem 1. In the following we use notations and terminologies in [2]. We have the following:

Lemma 1. *Fix $\beta\in[1,3)$. Suppose that $u_m\in C^\infty(\omega)$ satisfies*

$$(-\Delta+m')u_m(x)=0, \qquad x\in\omega$$
$$u_m(x)=0, \qquad x\in\partial\omega\cap\Gamma$$

and

$$\max\{|u_m(x)|; x\in\partial\boldsymbol{B}_r\cap\partial\omega\}=M_r(m),$$

$r=1,\cdots,\tilde{m}$. If $\partial\boldsymbol{B}_r\cap\partial\omega=\phi$, then we put $M_r(m)=0$. Under the above assumption, there exists a constant C independent of m such that

$$\|u\|_{L^2(\omega)}\leq C(\alpha/m)(m')^{-1/4}\Big(\sum_{r=1}^{\tilde{m}} M_r(m)^2\Big)^{1/2}$$
$$+C(\alpha/m)(m')^{-1/4}\Big(\sum_{\substack{r,s=1\\ r\neq s}}^{\tilde{m}}\exp(-(m')^{1/2}|w_r-w_s|)M_r(m)M_s(m)\Big)^{1/2}$$

holds.

By using Lemma 1 we get the following ;

Proposition 1. *Fix* $\beta \in [1,3)$ *and* $\varepsilon > 0$. *Then, we can take* $\nu > 0$, $\gamma \in (\beta - 1, 2)$ *such that*

$$\lim_{m\to\infty} \boldsymbol{P}(w(m) \in \Omega^{\tilde{m}}\,;\, m^{2\gamma-(\beta-1)+\nu}\|\boldsymbol{Q}_{(m')}\|_{L^2(\omega)} \leq \varepsilon) = 1$$

holds.

We have the following probabilistic result.

Proposition 2. *Let* $\{u_m(w(m))\}_{m=1}^{\infty}$ *be a sequence of* $L^2(\Omega)$*-valued random variables on* $\Omega^{\tilde{m}}$ *such that* $\|u_m(w(m))\|_{L^2(\Omega)} \leq 1$. *Fix* $\beta \in [1,3)$. *Then we can take* $\tilde{\nu} > 0$, $\gamma \in (\beta - 1, 2)$ *such that*

$$\lim_{m\to\infty} \boldsymbol{P}_m(m^{2\gamma-(\beta-1)+\tilde{\nu}}\|(\boldsymbol{H}_{(m')} - \tilde{\boldsymbol{G}}_{(m')})(u_m(w(m))\|_{L^2(\Omega)} \leq \varepsilon) = 1$$

holds for any $\varepsilon > 0$.

By using the same argument as in Ozawa [2] we get Theorem 1.

The readers may refer to Kac [1], Rauch-Taylor [5] Papanicolaou-Varadhan [4], Simon [6] and the literatures cited there for related topics.

References

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