Let $\mathfrak{U}$ be the subgroup of $\mathfrak{R}(f)$ which corresponds to K. Then the elliptic unit η of K is defined by the following:

$$\eta=\prod_{\mathfrak{k}\in\mathfrak{U}}\sqrt{\mathrm{Im}\,(\gamma_{\mathfrak{r}\mathfrak{k}})\,\mathrm{Im}\,(\gamma_{\mathfrak{r}^3\mathfrak{k}})/\mathrm{Im}\,(\gamma_{\mathfrak{k}})\,\mathrm{Im}\,(\gamma_{\mathfrak{r}^2\mathfrak{k}})}\,|\eta(\gamma_{\mathfrak{r}\mathfrak{k}})\eta(\gamma_{\mathfrak{r}^3\mathfrak{k}})/\eta(\gamma_{\mathfrak{k}})\eta(\gamma_{\mathfrak{r}^2\mathfrak{k}})|^2.$$

Here $\eta(z)$ is the Dedekind eta function, and $\gamma_{\mathfrak{k}}$ is a complex number with positive imaginary part such that $Z\gamma_{\mathfrak{k}}+Z$ belongs to the class $\mathfrak{k}\in\mathfrak{R}(f)$. The class $\mathfrak{r}\in\mathfrak{R}(f)$ is chosen so that $\mathfrak{r}\mathfrak{U}$ generates the cyclic quotient group $\mathfrak{R}(f)/\mathfrak{U}$. The definition of η is independent of the choice of $\gamma_{\mathfrak{k}}$ and $\mathfrak{r}$. Therefore, if $\mathfrak{R}(f)$ and $\mathfrak{U}$ are explicitly given, we can calculate an approximate value of η using Lemma 3 of [2].

It is possible to obtain $\mathfrak{R}(f)$ and $\mathfrak{U}$ explicitly, although it seems to be very complicated in the actual calculation.

§6. Appendix. (i) The following propositions help to determine ε_2 and ε_3.

Proposition 2. (i) *Assume h_2 or h_3 is odd. Then $\varepsilon_3\neq\eta_3$ if $\sqrt{\eta}$ does not belong to K.* (ii) *Assume h_2 or h_3 is prime to 3. Then $\varepsilon_2\neq\eta_2$ if $\sqrt[3]{\eta}$ does not belong to K.*

Proposition 3. *Let f and d be as in §5, and let d_2 be the discriminant of K_2. Assume $\sqrt[3]{\eta_2}$ belongs to K. Then $d=3d_2$ or $3d_2=d$; and f is a power of 3.*

(ii) The galois closure L of K/Q contains a totally imaginary sextic subfield K' not conjugate to K. Further algorithm to compute the class number and fundamental units of K' exists. It uses the results in [1].

Corrections to References [2] and [3]. In [2], we add the assumption that "$D\neq-23$" throughout the note. See also [4] in detail. In Proposition 6 of [3], for '$\sqrt{\eta_e}$' read "$\sqrt{\eta_2}$". In the definition of H_+ in [3], line 6 of §1, for 'positive units' read "positive relative units".

References

[1] K. Nakamula: A construction of the groups of units of some number fields from certain subgroups (preprint).

[2] ——: Class number calculation and elliptic unit. I. Proc. Japan Acad., **57**A, 56–59 (1981).

[3] ——: Class number calculation and elliptic unit. II. ibid., **57**A, 117–120 (1981).

[4] ——: Class number calculation of a cubic field from the elliptic unit (to appear in J. reine angew. Math.).

[5] R. Schertz: Über die Klassenzahl gewisser nicht galoisscher Körper 6-ten Grades. Abh. Math. Sem. Hamburg., **42**, 217–224 (1974).