27. *Zeta Functions in Several Variables Associated with Prehomogeneous Vector Spaces. II*[*)]

A Convergence Criterion

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(Communicated by Shokichi IYANAGA, M. J. A., Feb. 12, 1981)

This article is a continuation of [2]. Here we present a sufficient condition of the convergence of zeta functions introduced in the previous paper. We keep the notation and the assumptions in [2] except (A.3).

5. Let H be the identity component (in the Zariski topology) of the group

$$\{g \in G;\ \chi(g)=1 \qquad \text{for all } \chi \in X_\rho(G)\}.$$

We consider the following four conditions:

(I) *For any* $t=(t_1, \cdots, t_n) \in (C^\times)^n$, *$H$ acts transitively on*

$$V(t)=\{x \in V-S;\ P_i(x)=t_i\ (1 \leq i \leq n)\}.$$

(S) *The group*

$$H_x=\{g \in H;\ \rho(g)x=x\}$$

is a connected semi-simple algebraic group for any $x \in V-S$.

(W) *For any* $x \in V_{\boldsymbol{Q}}-S_{\boldsymbol{Q}}$, *the Tamagawa number of* H_x *does not exceed some positive constant independent of* x.

(H) *For any* $x \in V_{\boldsymbol{Q}}-S_{\boldsymbol{Q}}$ *and for any inner* $\boldsymbol{Q}$*-form* A *of* H_x, *the canonical mapping*

$$H^1(\boldsymbol{Q}, \tilde{A}) \longrightarrow \prod_v H^1(\boldsymbol{Q}_v, \tilde{A})$$

is bijective where $\tilde{A}$ *is the universal covering group of* A *defined over* $\boldsymbol{Q}$ *and the product is over all places of* $\boldsymbol{Q}$.

Theorem 4. *If* (G, ρ, V) *satisfies the conditions* (A.1), (I), (S), (W), *and* (H), *then* (G, ρ^*, V^*) *also satisfies the conditions* (I), (S), (W) *and* (H). *Moreover the integrals* $Z(f, L; s)(f \in \mathcal{S}(V_{\boldsymbol{R}}))$ *and* $Z^*(f^*, L^*; s)(f^* \in \mathcal{S}(V_{\boldsymbol{R}}^*))$ *are absolutely convergent when* $\mathrm{Re}\, s_1, \cdots, \mathrm{Re}\, s_n$ *are sufficiently large.*

Theorem 5. *Further assume that every* $\boldsymbol{Q}$*-irreducible component of* S *is absolutely irreducible. Then* $Z(f, L; s)$ *(resp.* $Z^*(f^*, L^*; s)$*) is absolutely convergent for* $\mathrm{Re}\, s_1 > \delta_1, \cdots, \mathrm{Re}\, s_n > \delta_n$ *(resp.* $\mathrm{Re}\, s_1 > \delta_1^*, \cdots, \mathrm{Re}\, s_n > \delta_n^*$*).*

Remarks. (1) The condition (S) implies the condition (A.2).

*) Supported by the Grant in Aid for Scientific Research of the Ministry of Education No. 574050.

(2) If H_x has no simple component of type E_8, the condition (S) implies the condition (H). The so-called Weil conjecture asserts that the Tamagawa number of any simply connected algebraic group defined over $\boldsymbol{Q}$ is equal to 1. This conjecture is established for a fairly wide class of semi-simple algebraic groups. These observations show that the conditions (H) and (S) are not so restrictive. The other conditions are rather easy to check.

(3) If the group H_x is trivial for $x \in V-S$, then we may consider that (G, ρ, V) satisfies (S), (W) and (H).

We indicate the idea of the proof of our theorems. By the assumptions, we can reduce the convergence of $Z(f, L; s)$ to that of an integral of the following type:

$$\int_{(V-S)_A} \prod_{i=1}^{n} |P_i(x)|_A^{s_i} f(x) | \lambda^{-1} dx|_A$$

where we use the standard notation in adele geometry and f is a Schwartz-Bruhat function on V_A. We are able to handle such an integral by the technique developed in T. Ono [1]. Details will appear elsewhere.

References

[1] T. Ono: An integral attached to a hypersurface. Amer. J. Math., **90**, 1224–1236 (1968).

[2] F. Sato: Zeta functions in several variables associated with prehomogeneous vector spaces. I. Functional equations. Proc. Japan Acad., **57**A, 74–79 (1981).