5. *On the Microlocal Structure of a Regular Prehomogeneous Vector Space Associated with GL(8)*

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Let $V(n)$ be the n-dimensional vector space over C spanned by $u_1, \cdots, u_n$. Then the general linear group $GL(n)$ acts on $V(n)$ by $\rho_1(g)(u_1, \cdots, u_n)=(u_1, \cdots, u_n)g$ for $g \in GL(n)$.

Let V be the vector space spanned by skew-tensors $u_i \wedge u_j \wedge u_k$ $(1 \leq i<j<k \leq n)$ of degree three. Then the action $\rho=\Lambda_3$ of $GL(n)$ on V is given by $\rho(g)(u_i \wedge u_j \wedge u_k)=\rho_1(g)u_i \wedge \rho_1(g)u_j \wedge \rho_1(g)u_k$. The triplet $(GL(n), \Lambda_3, V)$ is a regular prehomogeneous vector space if and only if $n=3, 6$, 7 or 8 (see [1]). For the case $n=3$, 6 or 7, its microlocal structure has been investigated in [2]. In this article, we study the remaining case, i.e., $n=8$. We use the same notations as in [3].

A brief sketch of the present article and [3] had been given in [6].

§ **1. The orbits.** The orbital decomposition of this space $(GL(8), \Lambda_3, V)$ was completed by Gurevich (see [4]). A representative point of each orbit is given in Table I.

Table I. Representative points of the orbits and their isotropy subgroups

Numbers	Representative points	Isotropy subgroups
0, 56	$123+147+148+257+368+456$	$SL(3)$
1, 40	$4\langle 148\rangle-8\langle 157\rangle-2\langle 238\rangle+247$ $+4\langle 256\rangle-2\langle 346\rangle$	$(SL(2)\times GL(1))\cdot(G_a)^5$
3, 31	$138+167+247-256+345$	$(SL(2)\times GL(1)^2)\cdot U(6)$
4, 25	$136+147+236-258-345$	$GL(1)^3\cdot U(9)$
6, 21	$127-156+236-245-348$	$(SL(2)\times GL(1)^2)\cdot U(9)$
8, 24	$134+156+234+278$	$(SL(2)^3\times GL(1))\cdot(G_a)^6$
8, 16	$128+147-156-237+246+345$	$(SL(2)\times GL(1))\cdot U(12)$
9, 18	$136-145+234+278$	$(SL(2)^2\times GL(1)^2)\cdot U(9)$
10, 13	$128-137+156-246+345$	$(SL(2)\times GL(1)^2)\cdot U(13)$
12, 12	$136+147-235+248$	$(SL(2)^2\times GL(1)^2\cdot(G_a)^{12}$
13, 10	$128-137+146+236-245$	$(SL(2)\times GL(1))\cdot U(17)$
14, 28	$125+136+147+234+567$	$(G_2\times GL(1))\cdot(G_a)^7$
15, 15′	$157+168+234$	$(SL(3)\times Sp(2)\times GL(1))\cdot(G_a)^4$
15′, 15	$127+136+246+345$	$(SL(2)^2\times GL(1)^2)\cdot U(15)$
16, 8	$128-137+156+234$	$(SL(2)^2\times GL(1)^2)\cdot U(16)$
18, 9	$127+134-256$	$(SL(2)^2\times GL(1)^3)\cdot U(17)$

21, 6	125+136+147+234	$(SL(3)\times GL(1)^2)\cdot U(19)$
24, 8	123+456	$(SL(3)^2\times SL(2)\times GL(1))\cdot (G_a)^{12}$
25, 4	126+135−234	$(SL(3)\times SL(2)\times GL(1)^2)\cdot U(20)$
28, 14	125+136+147	$(Sp(3)\times GL(1)^2)\cdot U(13)$
31, 3	124+135	$(SL(3)\times Sp(2)\times GL(1)^2)\cdot U(19)$
40, 1	123	$(SL(5)\times SL(3)\times GL(1))\cdot (G_a)^{15}$
56, 0	0	$GL(8)$

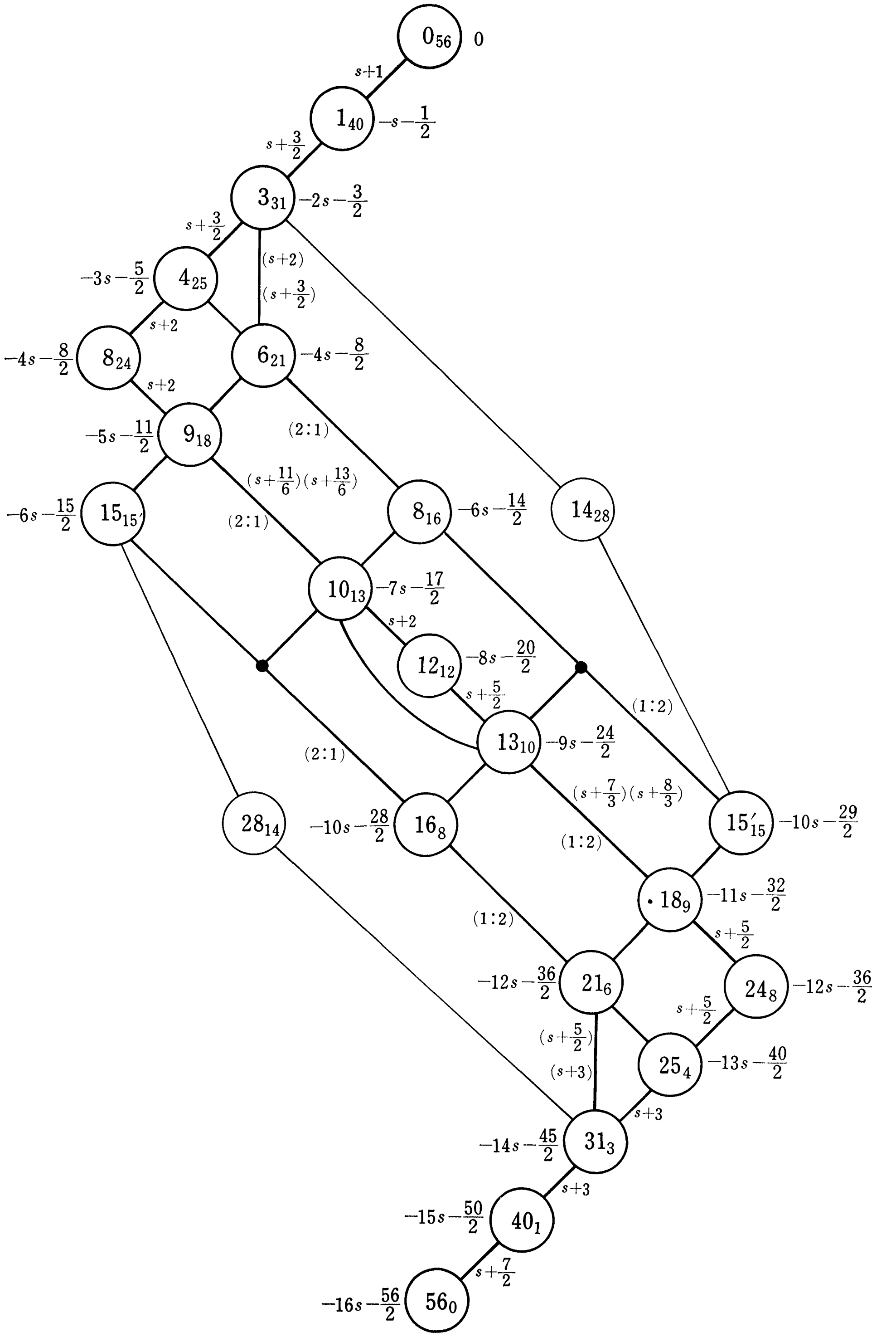
Remark 1.1. In Table I, $i\ j\ k$ stands for $u_i\wedge u_j\wedge u_k$ $(1\leq i<j<k\leq 8)$.

Remark 1.2. The isotropy subgroup of each orbit is given in Table I up to a local isomorphism. We use the following conventions; for example, $(SL(2)\times GL(1))\cdot U(12)$ stands for a semi-direct product of the reductive group $SL(2)\times GL(1)$ and a 12-dimensional unipotent group. G_a denotes the one dimensional additive group.

Remark 1.3. In Table II, we list the representative points in

Table II

Numbers in Table I	Numbers in [4]	Representative points in [4]
0, 56	XXIII	123+145+246+278+347+368+567
1, 40	XXII	123+145+246+278 +368+567
3, 31	XXI	123+145 +278 +368+567
4, 25	XX	145+246+278+347+368+567
6, 21	XVIII	145+246+278 +368+567
8, 24	XIX	145+246+278+347+368
8, 16	XV	123+145+246 +347+368+567
9, 18	XVII	145+246+278 +368
10, 13	XIV	123+145+246 +368+567
12, 12	XIII	123+145 +368+567
13, 10	XII	145+246 +347+368+567
14, 28	X	123+145+246 +347 +567
15, 15′	XVI	145 +278 +368
15′, 15	IX	123+145+246 +567
16, 8	XI	145+246 +347+368
18, 9	VIII	123+145 +567
21, 6	VII	145+246 +347 +567
24, 8	V	123 +456
25, 4	IV	156+246 +345
28, 14	VI	145+246 +347
31, 3	III	145+246
40, 1	II	567
56, 0	I	0



0_56 0
s+1
1_40 −s−1/2
s+3/2
3_31 −2s−3/2
s+3/2
4_25 −3s−5/2
(s+2)
(s+3/2)
s+2
8_24 −4s−8/2
6_21 −4s−8/2
s+2
9_18 −5s−11/2
(2:1)
(s+11/6)(s+13/6)
15_15′ −6s−15/2
(2:1)
8_16 −6s−14/2
14_28
10_13 −7s−17/2
s+2
12_12 −8s−20/2
s+5/2
(1:2)
13_10 −9s−24/2
(2:1)
(s+7/3)(s+8/3)
28_14
16_8 −10s−28/2
15′_15 −10s−29/2
(1:2)
18_9 −11s−32/2
(1:2)
s+5/2
21_6 −12s−36/2
24_8 −12s−36/2
s+5/2
(s+5/2)
(s+3)
25_4 −13s−40/2
s+3
31_3 −14s−45/2
s+3
40_1 −15s−50/2
s+7/2
56_0 −16s−56/2

Fig. 1

Gurevich [4]. Our choice of the representative points in Table I is suitable to obtain the isotropy subgroups in a simple form.

In [4] the eight linearly independent vectors are denoted by a, b, c, p, q, r, s, t. In Table II, however, they are denoted by $1, 2, 3, \cdots, 8$ according to our convention.

Remark 1.4. Representative points of (24, 8) and (25, 4) can be taken $123+567$ and $246+347+567$, respectively.

§ 2. The holonomy diagram. We give the holonomy diagram in Fig. 1. For its definition, see [5].

Remark 2.1. In Fig. 1, we show the following data for each good holonomic variety Λ.

(1) The order $\mathrm{ord}_\Lambda f^s = -ms - n/2$ of the simple holonomic system $\mathcal{M}_s = \mathcal{E} f^s$ where $\mathcal{E}$ denotes the sheaf of micro-differential operators.

(2) The intersection exponent $(\mu : \nu)$, when it is not indefinite.

(3) The ratio $b_{\Lambda'}(s)/b_\Lambda(s)$ of the local b-functions $b_{\Lambda'}(s)$ and $b_\Lambda(s)$ when Λ and Λ' have a one-codimensional intersection. Those ratios corresponding to the opposite sides of each rectangle are the same.

Remark 2.2. The conormal bundle of the orbit (14, 28) or (28, 14) is not prehomogeneous.

§ 3. The *b*-function. Proposition 3.1. *The b-function $b(s)$ of the triplet $(GL(8), \Lambda_3, V)$ is given by*

$$b(s)=(s+1)\left(s+\frac{3}{2}\right)^2\left(s+\frac{11}{6}\right)(s+2)^3\left(s+\frac{13}{6}\right)\left(s+\frac{7}{3}\right)\left(s+\frac{5}{2}\right)^3 \times\left(s+\frac{8}{3}\right)(s+3)^2\left(s+\frac{7}{2}\right).$$

Remark 3.2. We have obtained the above results by the method in [5].

References

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