9. On the Microlocal Structure of the Regular Prehomogeneous Vector Space Associated with $SL(5)\times GL(4)$. I

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In this article, we study the microlocal structure of the triplet $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1, V(10)\otimes V(4))$. In the present article, we give its orbital decomposition, the main part of its holonomy diagram, and some of its local b-functions. Incidentally, this prehomogeneous vector space is the most intricate of all of the reduced irreducible prehomogeneous vector spaces. (See [1].)

Notations. $G=SL(5)\times GL(4)$, $\rho=\Lambda_2\otimes\Lambda_1$, $V=V(10)\otimes V(4)$, $V^*=$the dual space of V, $G_x=$the isotropy subgroup of G at x, $\mathfrak{g}=$the Lie algebra of G, $\mathfrak{g}_x=$the Lie algebra of G_x, i.e. the isotropy subalgebra at x, $V_x^*=$the conormal vector space $(d\rho(\mathfrak{g})x)^\perp$ at x, $b(s)=$the b-function of (G,ρ,V), $b_{ij}(s)=$the local b-function of the holonomic variety Λ_{ij} which is the conormal bundle of the G-orbit S_{ij} defined as follows. When the conormal bundle $\Lambda(\subset V\times V^*)$ of a G-orbit S in V coincides with the conormal bundle $\Lambda^*(\subset V\times V^*)$ of a G-orbit S^* in V^*, we say that they are each other's dual orbits. We denote by S_{ij} the i-codimensional G-orbit in V whose dual orbit is j-codimensional.

§1. The orbital decomposition. Proposition 1.1. *The triplet* $(SL(5)\times GL(4), \Lambda_2\otimes\Lambda_1, V(10)\otimes V(4))$ *has* 62 *orbits given in Table* I.

Remark 1.2. The fact that these 62 orbits are mutually distinct follows immediately by comparing their isotropy subalgebras at representative points (see Table II and Remark 1.5). In order to show that no other orbit exists, we need to classify the zeros of the localization of the relative invariant on each of several holonomic varieties. Note that this space has a relatively invariant irreducible polynomial $f(x)$ of degree 40, which is unique up to a constant multiple (see [1]). The 62 orbits are shown in Table I. Each of the 33 rows lists a pair of mutually dual orbits. Note that each of the four orbits $S_{7,7}$, $S_{8,8}$, $S_{9,9}$, and $S_{10,10}$ is dual to itself.

Remark 1.3. As a matter of notation, the following method of abbreviation is used: for example, $236-137+128+459$ stands for $(u_2\wedge u_3)\otimes u_6-(u_1\wedge u_3)\otimes u_7+(u_1\wedge u_2)\otimes u_8+(u_4\wedge u_5)\otimes u_9$ $(\in V(10)\otimes V(4))$, where the $u_i\wedge u_j$ $(1\leqslant i<j\leqslant 5)$ and u_k $(6\leqslant k\leqslant 9)$ span $V(10)$ and $V(4)$ respectively.

Remark 1.4. Let (G_x, ρ_x, V_x^*) be the localization of (G, ρ, V) at x induced by the natural action of G_x on V_x^*. We define its generic points y of V_x^* by the condition that x is a point of the least codimensional G_y-orbit in the conormal vector space $(V^*)_y^*$, even when V_x^* is not a prehomogeneous vector space (abbreviated N.P. in Table I).

Remark 1.5. The isotropy subgroups of each orbit is locally isomorphic to the group of the orbit in Table II. We use the following conventions: for example, $(GL(1)\times GL(1))\cdot U(4)$ stands for the semi-direct product of the reductive group $GL(1)\times GL(1)$ and a 4-dimensional unipotent group to which the isotropy subgroup of the orbit $S'_{6,14}$ is locally isomorphic. Although three pairs of orbits such as $S'_{6,14}$ and $S''_{6,14}$ have apparently the same isotropy subgroups in Table II, one can show by looking at the isotropy subalgebras that they can not be transformed by any inner automorphism, and hence they are distinct orbits.

§2. The holonomy diagram. Since the entire holonomy diagram of this space is too complicated to draw here, we give the main part of the diagram in Fig. 1.

Remark 2.1. With regard to Fig. 1, we have calculated the following data for each good holonomic variety Λ. (See [2].)

(1) The order $\mathrm{ord}_\Lambda f^s$ of the simple holonomic system $\mathcal{M}_s=\mathcal{E}f^s$, where $\mathcal{E}$ denotes the sheaf ofm icro-differential operators. In this case, we have $\mathrm{ord}_\Lambda f^s=-ms-m/2$ for some positive integer m.

(2) Let $g_1, \cdots, g_l$ be the relatively invariant irreducible polynomials of the localization (G_x, ρ_x, V_x^*) with the corresponding infinitesimal characters $\delta\chi_1, \cdots, \delta\chi_l$, where Λ is a conormal bundle of $\rho(G)x$. Then there exist two relative invariants corresponding to $-\delta\chi$ and $2\,\mathrm{tr}_{V_x^*}$ where $\delta\chi$ is the character of f. We shall calculate a_j and c_j $(1\leqslant j\leqslant l)$ satisfying $-\delta\chi=\sum_{j=1}^{l} c_j\delta\chi_j$ and $\mathrm{tr}_{V_x^*}=\sum_{j=1}^{l} a_j\delta\chi_j$.

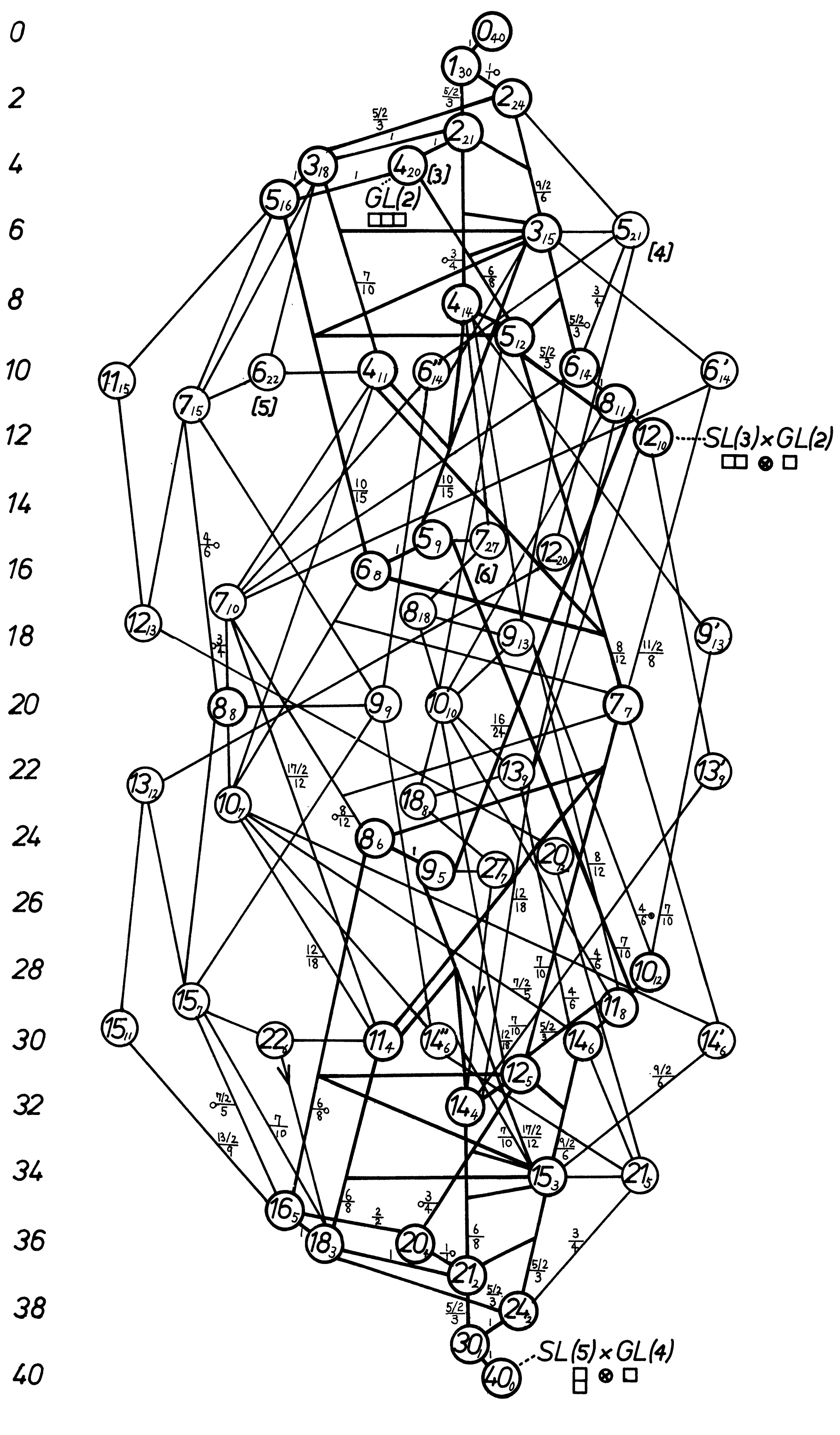
(3) The intersection exponent $(\mu:\nu)$.

Remark 2.2. (1) In Fig. 1, we have indicated the orders $(-ms-m/2)$ by the letter m on the left column and arranged each of the good holonomic varieties according to their orders.

(2) For each g_j $(1\leqslant j\leqslant l)$ in Remark 2.1, there exists a holonomic variety Λ_j satisfying $\mathrm{codim}\ \Lambda\cap\Lambda_j=1$. We write a_j/c_j beside the line connecting Λ and Λ_j. And when $\Lambda_j=\Lambda_{j'}$ for two j and j' (resp., $\Lambda_j=\Lambda_{j'}=\Lambda_{j''}$ for three j, j' and j''), we add a circle at the end of the fraction line, i.e.—○ (resp., —⦿).

§3. The factors of the *b*-function. We show some results concerning the b-function.

Proposition 3.1. *The local b-function of $\Lambda_{12,10}$ coincides with the b-function of the irreducible regular prehomogeneous vector space $(SL(3)\times GL(2), 2\Lambda_1\otimes\Lambda_1, V(6)\otimes V(2))$, i.e.*



0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
GL(2)
[3]
[4]
[5]
[6]
SL(3)×GL(2)
SL(5)×GL(4)

Fig. 1.

$$b_{12,10}(s)=(s+1)^4(s+5/6)^2(s+7/6)^2(s+3/4)^2(s+5/4)^2.$$

Proposition 3.2. *We have* $b_{6,8}(s)/b_{5,9}(s)=(s+1)$, $b_{3,18}(s)/b_{2,24}(s)=(s+5/6)(s+7/6)$ *and* $b_{3,18}(s)/b_{2,21}(s)=(s+1)$.

Remark 3.3. We have obtained the above results by the Kashiwara-Oshima formula. But this formula is not sufficient for calculating the remaining factors. For this, it is necessary to employ a variety of other methods. The details of the results will be given in a coming article.

Table I. Representative points of orbits

	orbits		*dual orbits*	
numbers	*representative points*		*representative points*	*numbers*
0,40	256−346+157−247−148+238 −129+459]		0	40,0
1,30	456+137+257+148+238+159 +249		126	30,1
2,24	456+346+137+148+238+259		136−127	24,2
2,21	126+456+157+257+238+248 +139		146−236−127	21,2
3,18	126+356−157+148+238+249		146+236−137+128	18,3
3,15	156+246+346+147+237+138 +259		156−246−137+128	15,3
4,20	256−157+348+129+149−239		126+347	20,4
4,14	156+246+147+237+128+359		256−346−137+128	14,4
4,11	256+346+157+247+148+238 +139		156−246+147+237−138 +129	11,4
5,21	346+137+247+128+259+459	N.P.	126+137+148	21,5
5,16	156+257+348+139+249		246−137+128	16,5
5,12	146−236+256−157+128+349		156+246−147−237−138	12,5
5,9	256−346+157−247−148+238 −129		256−346+147+237−138 +129	9,5
6,22	136+147+237+158+248+259	N.P.	236−137+128	22,6
6,14	236+356−157+248+139		456+137−128	14,6
(6,14)′	156+246+147+357+138+129	N.P.	136−246−147+257+128	(14,6)′
(6,14)″	346−157+247−148+238+129	N.P.	156+236−147+138+129	(14,6)″
6,8	256−346+147−237−158+129		256−346+147−138+238 +129	8,6
7,27	346−2⟨256⟩+8⟨157⟩−247 −2⟨148⟩+238	N.P.	136+246	27,7
7,15	346−157−148+238+129	N.P.	146+237−138+129	15,7
7,10	146−256−347−128−139	N.P.	136−246−157+238+129	10,7
7,7	256−346−157+247−138+129		self-dual	7,7
8,18	146+256−157−347−238	N.P.	156+236+147	18,8
8,11	146+236−137+128+459		146−236−127+358	11,8
8,8	246+357+128+139		self-dual	8,8
9,13	256+346−147−247+138	N.P.	236+456−137+128	13,9
(9,13)′	136−247+158−258−129	N.P.	156−246+237−138+129	(13,9)′
9,9	156+346+147−238−129	N.P.	self-dual	9,9
10,10	256+346−147−237+138	N.P.	self-dual	10,10
11,15	256+346−147+138+129	N.P.	346+137+247+128	15,11
12,20	136−237−148+249	N.P.	126+137+148+159	20,12

12,13	236−247−158+129	N.P.	246−147−237+138+129	13,12
12,10	236−137+128+459		136+157+257+248	10,12

Table II. The isotropy subgroups

numbers	*groups*	*groups*	*numbers*
0,40	$\{e\}$	$SL(5)\times GL(4)$	40,0
1,30	$GL(1)$	$(SL(2)\times GL(3)\times GL(3))\cdot U(9)$	30,1
2,24	$GL(1)\times GL(1)$	$(SL(2)\times GL(1)\times SL(2)\times GL(1)\times GL(2))\cdot U(12)$	24,2
2,21	$GL(1)\cdot U(1)$	$(SL(2)\times GL(1)\times GL(1)\times GL(2))\cdot U(12)$	21,2
3,18	$(GL(1)\times GL(1))\cdot U(1)$	$(SL(2)\times GL(1)\times GL(1)\times GL(1))\cdot U(12)$	18,3
3,15	$GL(1)\cdot U(2)$	$(GL(1)\times GL(1)\times GL(1)\times GL(1))\cdot U(11)$	15,3
4,20	$SL(2)\times GL(1)$	$(SL(2)\times GL(1)\times SL((2))\times GL(1)\times GL(2))\cdot U(8)$	20,4
4,14	$(GL(1)\times GL(1))\cdot U(2)$	$(SL(2)\times GL(1)\times GL(1)\times GL(1))\cdot U(8)$	14,4
4,11	$GL(1)\cdot U(3)$	$(GL(1)\times GL(1))\cdot U(9)$	11,4
5,21	$(SL(2)\times GL(1))\cdot U(1)$	$(SL(3)\times GL(1)\times GL(1)\times GL(1))\cdot U(10)$	21,5
5,16	$SL(2)\times GL(1)\times GL(1)$	$(GL(1)\times GL(1)\times GL(1)\times GL(1)\times GL(1))\cdot U(11)$	16,5
5,12	$(GL(1)\times GL(1))\cdot U(3)$	$(GL(1)\times GL(1)\times GL(1))\cdot U(9)$	12,5
5,9	$GL(1)\cdot U(4)$	$(GL(1)\times GL(1))\cdot U(7)$	9,5
6,22	$(SL(2)\times GL(1))\cdot U(2)$	$(SL(3)\times GL(2)\times GL(1))\cdot U(9)$	22,6
6,14	$(GL(1)\times GL(1)\times GL(1))\cdot U(3)$	$(SL(2)\times GL(1)\times SL(2)\times GL(1)\times GL(1))\cdot U(5)$	14,6
(6,14)′	$(GL(1)\times GL(1))\cdot U(4)$	$(SL(2)\times GL(1)\times GL(1))\cdot U(9)$	(14,6)′
(6,14)″	$(GL(1)\times GL(1))\cdot U(4)$	$(SL(2)\times GL(1)\times GL(1))\cdot U(9)$	(14,6)″
6,8	$(GL(1)\times GL(1))\cdot U(4)$	$(GL(1)\times GL(1))\cdot U(6)$	8,6
7,27	$(SL(2)\times GL(1))\cdot U(3)$	$(Sp(2)\times GL(1)\times GL(3))\cdot U(7)$	27,7
7,15	$(GL(1)\times GL(1)\times GL(1))\cdot U(4)$	$(SL(2)\times GL(1)\times GL(1)\times GL(1))\cdot U(9)$	15,7
7,10	$(GL(1)\times GL(1)\times GL(1))\cdot U(4)$	$(GL(1)\times GL(1)\times GL(1))\cdot U(7)$	10,7
7,7	$(GL(1)\times GL(1))\cdot U(5)$		7,7
8,18	$(GL(1)\times GL(1)\times GL(1))\cdot U(5)$	$(SL(2)\times GL(1)\times GL(1)\times GL(2))\cdot U(9)$	18,8
8,11	$(SL(2)\times GL(1)\times GL(1))\cdot U(3)$	$(GL(1)\times GL(1)\times GL(1)\times GL(1))\cdot U(7)$	11,8
8,8	$(GL(1)\times GL(1)\times GL(1)\times GL(1))\cdot U(4)$		8,8
9,13	$(GL(1)\times GL(1)\times GL(1))\cdot U(6)$	$(SL(2)\times SL(2)\times GL(1)\times GL(1))\cdot U(5)$	13,9
(9,13)′	$(GL(1)\times GL(1)\times GL(1))\cdot U(6)$	$(SL(2)\times GL(1)\times GL(1))\cdot U(8)$	(13,9)′
9,9	$(GL(1)\times GL(1)\times GL(1))\cdot U(6)$		9,9
10,10	$(GL(1)\times GL(1)\times GL(1))\cdot U(7)$		10,10
11,15	$(SL(2)\times GL(1)\times GL(1))\cdot U(6)$	$(GL(2)\times GL(2))\cdot U(7)$	15,11
12,20	$(SL(2)\times GL(1)\times SL(2)\times GL(1))\cdot U(4)$	$(SL(4)\times GL(1))\cdot U(4)$	20,12
12,13	$(SL(2)\times GL(1)\times GL(1)\times GL(1))\cdot U(6)$	$(SL(2)\times GL(1)\times GL(1))\cdot U(8)$	13,12
12,10	$SL(3)\times GL(2)$	$(GL(1)\times GL(1)\times GL(1)\times GL(1))\cdot U(6)$	10,12

References

[1] M. Sato and T. Kimura: A classification of irreducible prehomogeneous vector spaces and their relative invariants. Nagoya Math. J., 65, March, 1–155 (1977).

[2] M. Sato, M. Kashiwara, and T. Kimura: Microlocal Analysis of Prehomogeneous Vector Spaces (in preparation).