

### 38. Decomposition of the Unitary Representations of $SL(2, C)$ Induced by the Discrete Series of $SU(1, 1)$

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Let  $G=SL(2, C)$ ,  $G_0$  be one of subgroups  $SU(1, 1)$ ,  $SU(2)$  or  $E(2)$ , and  $\pi$  be an irreducible unitary representation of  $G_0$ . Denote  $\text{Ind}_{G_0 \uparrow G} \pi$  the representation of  $G$  induced by  $\pi$ . We consider the problem to decompose  $\text{Ind}_{G_0 \uparrow G} \pi$  to a direct integral of irreducible representations. Though many investigations are done about this problem by physicists, large part remains still open.

In this paper we shall treat this problem in the case  $G_0=SU(1, 1)$  and  $\pi=T_l^\pm$  (so-called representations of discrete series of  $G_0$ ). We refer to [1] for  $T_l^\pm$  ( $l=-1/2, -1, -3/2, \dots$ ) and to [2] for irreducible representations  $\mathfrak{S}_{m, \rho}$  of the continuous series of  $SL(2, C)$ .

Our main result is as follows.

**Theorem.**  $\text{Ind}_{G_0 \uparrow G} T_l^\pm \simeq \sum_{m=-2l, -2l+2, -2l+4, \dots} \oplus \int_{-\infty}^{\infty} \mathfrak{S}_{m, \rho} d\rho.$

We shall outline the proof. First we restrict  $\omega=\text{Ind}_{G_0 \uparrow G} T_l^\pm$  to the subgroup  $SU(2)$  of  $G$  and determine the space  $W_k$  spanned by highest weight vectors in  $T_k$  component (see [1]). This space  $W_k$  is characterized as an eigenspace of  $H_3$  and  $\Delta_0$  (see [2]):

$$H_3 f = k f, \quad \Delta_0 f = -k(k+1) f.$$

Since  $\mathfrak{S}_{m, \rho} | SU(2)$  contains  $T_k$  with the multiplicity one if and only if  $k-m/2$  is a nonnegative integer, the spectral measure of  $G$ 's Laplace-Beltrami operator  $\Delta'$  restricted to  $W_k \ominus F_+ W_{k-1}$  gives the distribution of  $\mathfrak{S}_{2k, \rho}$ -components (see [2] for  $F_+$  and  $\Delta'$ ). On account of the unitary equivalence between  $\Delta' | W_k$  and  $\Delta' | \overline{F_+ W_k}$  we can determine the spectral measure inductively using the spectral measure of  $\Delta' | W_k$ . Indeed the operator  $\Delta' | W_k$  turns out to be unitary equivalent to an ordinary differential operator of first order acting on vector-valued functions, hence we apply the Titchmarsh-Kodaira's expansion theorem to calculate the spectral measure of  $\Delta' | W_k$ . The details will be published elsewhere.

#### References

- [1] N. Ja. Vilenkin: Special functions and the theory of group representations. Amer. Math. Soc. Transl., **22** (1968).
- [2] M. A. Neumark: Lineare Darstellungen der Lorentzgruppe. VEB Deutscher Verlag der Wissenschaften, Berlin (1963).