41. *The Computation of the Path of a Ray and the Correction of the Aberrations of a Lens System. Part II.*

By Tatsuro SUZUKI.

Faculty of Engineering, Osaka University.

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The Correction of the Aberrations.

If it has been revealed as a result of trigonometric computations of the aberrations that the lens system has so large aberrations as unsuited to our purpose, we must correct them by changing slightly their curvatures and intervals of refracting surfaces or adopting another kind of glass. In the following section the method to obtain the suitable factor of modification of them will be discussed. For a while, let us regard the refractive index n to be a constant, which means that the kind of glass is not altered. As we shall be able to obtain easily the similar formulas even if n is regarded as a variable, the generality of the treatment is not disturbed by this simplification.

If the co-ordinates (h, θ) of an incident ray increase from (h, θ) to $(h+dh, \theta+d\theta)$, and the radius of the surface from r to $(r+dr)$, dh, $d\theta$ and dr being small fractions of h, θ and r respectively, then the co-ordinates of the refracted ray will be increased from (h', θ') to $(h'+dh', \theta'+d\theta')$.

Then, from (1) and (2) of the part I,

$$H' = \frac{1}{n}\frac{h}{h'}H = H, \tag{22}$$

$$d\theta' = A(R-H)+d\theta, \tag{23}$$

from (3) and (4)

$$H_{n+1} = B_n H_n + D_n K_n + E_n d\theta'_n, \tag{24}$$

$$d\theta_{n+1} = d\theta'_n, \tag{25}$$

where

$R = \dfrac{dr}{r}$: Factor of modification of radius.

$K = \dfrac{dC}{C}$: Factor of modification of the distance of the two consecutive centers of radius.

$H = \dfrac{dh}{h}, \quad H' = \dfrac{dh'}{h'}.$

$$A = \cot\beta - \cot\beta', \quad B_n = \frac{h'_n}{h_{n+1}}, \quad D_n - \frac{C_n \cdot \cos\theta'_n}{h_{n+1}},$$
$$E_n = \frac{C_n \cdot \sin\theta'_n}{h_{n+1}}. \qquad (26)$$

Substituting each value of h'_n, h_{n+1}, C_n, θ'_n, β and β', which have been obtained as a result of the computation of the path of the ray, in (26), each ones of A, B, D, and E are now obtained. If we repeat the above calculations from the first to the last surface k in succession and we will obtain H_K, $d\theta'_K$ concerning the last surface. (Put $dh_1 = 0$, $d\theta_1 = 0$; for the path of the incident ray to the first surface does not move from the first position even if r_1 has been changed.)

As mentioned previously,

$$s = \frac{h'_K}{\cos\theta'_K}. \qquad (27)$$

From this

$$\frac{ds}{s} = H_K + \tan\theta'_K \cdot d\theta'_K. \qquad (28)$$

Substituting the above H_K, $d\theta'_K$ in (28) and putting in order, we have

$$\frac{ds}{s} = a_1R_1 + a_2R_2 + \cdots + a_iK_1 + a_{i+1}K_2 + \cdots. \qquad (29)$$

It is much convenient to compute the magnitude of the coefficients a_1, a_2, ..., if we use the table which was made previously according to (22)~(28).

Substituting the magnitudes of (h, θ) in this table one after another, we shall have the magnitudes of a_1, a_2, ... directly.

Concerning the paraxial ray we have by the same procedure,

$$\frac{d\bar{s}}{\bar{s}} = b_1R_1 + b_2R_2 + \cdots + b_iK_1 + b_{i+1}K_2 + \cdots, \qquad (30)$$

similar to (29).

For the focal length we have

$$\frac{df}{f} = c_1R_1 + c_2R_2 + \cdots + c_iK_1 + c_{i+1}K_2 + \cdots. \qquad (31)$$

Above equations mean that if each of the radius or the surface distance is modified by R_1, R_2, ... and K_1, K_2, ... respectively, (viz. increased by dr_1, dr_2, ... and dc_1, dc_2, ...), s, $\bar{s}$ or f will increase by ds, ds or df defined by (29), (30) and (31). About other aberrations, such as astigmatism or distortion, similar formulas will be easily obtained.

If we compute a_1, a_2, ..., with the result of the tracing about line D, ds is written as ds_D. If it is about F line, it will be written as ds_F. Similar subscript will be applied to s, viz., if it is about D line it will be written as s_D^s.

Now we can proceed the correction of aberrations. If the image point A_D (Fig. 6) by a rim ray (about D line) does not coincide with the image point $\bar{A}_D$ by the paraxial ray, viz. $s_D \neq \bar{s}_D$, there exists spherical aberration. If the image point $\bar{A}_F$ (about F line) does not coincide with $\bar{A}_D$, there must exist chromatic aberration, and so on. To correct these aberrations, r and c should be modified so as these image points would coincide with each other. In consequence of these modifications, s_D and $\bar{s}_D$ will increase by ds_D and $d\bar{s}_D$ respectively, and if

$$d\bar{s}_D - ds_D = \Delta s_D\,, \tag{32}$$

where

$$\Delta s_D = s_D - \bar{s}_D\,,$$

is satisfied, A_D will coincide with $\bar{A}_D$.

If

$$d\bar{s}_D - ds_F = \Delta s_F\,, \tag{33}$$

where

$$\Delta s_F = s_F - \bar{s}_D\,,$$

is satisfied, A_F may be coincided with $\bar{A}_D$, with the same modification given above. These Δs_D or Δs_F has been obtained in consequence of the ray tracing.

Substituting ds and $d\bar{s}$ of (29) and (30) to (32) and (33), and putting in order, finally we will be able to have the simultaneous equations about variables R and K as in (34).

The similar results will be obtained as to other aberrations.

$$\left.\begin{array}{l} A_1R_1 + A_2R_2 + \cdots\cdots + A_iK_1 + A_{i+1}K_2 + \cdots\cdots = \Delta s_1 \\ B_1R_1 + B_2R_2 + \cdots\cdots + B_iK_1 + B_{i+1}K_2 + \cdots\cdots = \Delta s_2 \\ C_1R_1 + C_2R_2 + \cdots\cdots + C_iK_1 + C_{i+1}K_2 + \cdots\cdots = \Delta s_3 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \quad (34)^*$$

Let us assume the number of equations in (34) as n and that of variables R and K as m. If $m = n$, solving (34), we will be able to have the factors necessary to modify r and C, and this will be the very corrections desired. Above equations are valid when all of the modifications dr_1, dr_2 ... are the small fractions of r_1, r_2

... c_1, c_2, ... (then, also R and K should be small values such as 0.01 or at most 0.1). Accordingly even if only one of them, i.e., either R or K, solved from (34) becomes so large as 0.5 or 1, we cannot accept these values. In this case, it means that it is impossible to hope to obtain such a correction in this lens system. But even in this case, it will be expected to give some informations about the type of the lens system which we should employ to wish such corrections. In this case, we must find out small values of R and K from (34) by trial method. As the coefficient A, B, C, D, ... of (34) are given, it is not so difficult to find out adequate values of R and K which will fulfil (34) pretty well. In general, as the coefficient A, B, C, D, ... are small such as 0.5 or 2, and also we need not to calculate values smaller than 0.001 m.m. about aberrations, then the trial method becomes quite an easy one with the use of ordinary slide rule. The difference between left- and right-hand side, found by substitution of R and K in (34), means the residual aberrations.

When $m > n$, we can choose zero for some of R and K.

When $m < n$, we must find out adequate R and K by trial method. From (34), we are also able to know the allowable errors of r and c, separately, (or lens thickness t). For instance, if there is an error of 0.1% about r_1, it will impairs the aberations as much as 0.001 A_1, 0.001 B_1, and so on. On the contrary, if the allowable error of each aberration is previously given, we are able to obtain the tolerance of r and c (viz. R and K) easily by reversing the calculation. The author is greatly indebted to Professor Gunji Shinoda and Professor Hiroshi Yoshinaga for valuable advices and for many stimulating discussions on the subject.