115. *Probability-theoretic Investigations on Inheritance.* *XV_1. Detection of Interchange of Infants.*[1]

By Yûsaku KOMATU.

Department of Mathematics, Tokyo Institute of Technology and Department of Legal Medicine, Tokyo Medical and Dental University.

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1. Problem to be discussed and its basic postulate.

With increasing spread of institution of maternity hospitals, the chance of almost simultaneous deliveries at the same place will also increase. It would then be expected that two new-born infants are interchanged intentionally or accidentally. When such a case arises and even when two infants resemble indistinguishably, there can be cases where the interchange is detected by means of an inherited character.

We now take *a triple consisting of a child and its parents as a unit of consideration.* Let two such triples be presented under a suspicion of interchange of infants. If in at most one triple there is an inconsistency with respect to types, i.e., an apparent infant possesses a type not producible by patents of the triple, then the decision of interchange is possible. This fact is a basic postulate for our discussion in the present chapter.

We consider the probability of an event that the decision is possible under a supposition of actual interchange. But, as the following discussion will show, the supposition of interchange is unessential. The problem concerns rather, given two matings and their respective children, to determine the actual relationship.

2. Illustration by *MN* blood type.

We begin with the *MN* blood type as the most simple model

1) Y. Komatu, Probability-theoretic investigations on inheritance. I. Distribution of genes; II. Cross-breeding phenomena; III. Further discussions on cross-breeding; IV. Mother-child combinations; V. Brethern combinations; VI. Rate of danger in random blood transfusion; VII. Non-paternity problems; VIII. Further discussions on non-paternity; IX. Non-paternity problems concerning mother-children combinations; X. Non-paternity concerning mother-child-child combinations; XI. Absolute non-paternity; XII. Problem of paternity; XIII. Estimation of genotypes; XIV. Decision of biovular twins. Proc. Japan Acad., **27**(1951); I. 371-377; II. 378-383, 384-387; III. 459-465, 466-471, 472-477, 478-483; IV. 587-592, 593-597, 598-603, 605-610, 611-614, 615-620; V. 689-693, 694-699; **28**(1952), VI. 54-58; VII. 102-104, 105-108, 109-111, 112-115, 116-120, 121-125; VIII. 162-164, 165-168, 169-171; IX. 207-212, 213-217, 218-223, 224-229; X. 249-253, 254-258, 259-264; XI. 311-316, 317-322; XII. 359-364, 365-369; XIII. 432-437, 438-443; XIV. 444-449.

of consideration. There are 6 kinds of matings indifferent to the order, namely

$$M\times M,\ M\times N,\ M\times MN,\ N\times N,\ N\times MN,\ MN\times MN.$$

Hence, there are $6^2=36$ sorts of combinations between two pairs of matings. Among them, those where both pairs are of a coincident kind of mating are evidently out of question for the interchange problem. Further, in view of mutual symmetry we have only to investigate a half of the remaining 30 sorts of combinations.

For a while let the order of members in each mating be indifferent. Let the mating of the first triple be $M\times M$. Probability of the mating $M\times M$ is $\overline{M}^2=s^4$ and its true child is always of the type M. If the second mating is $M\times N$, then its probability is $2\overline{M}\overline{N}=2s^2t^2$ and its true child is always MN. Hence, the probability of an event that this combination of matings occurs and the detection of interchange is possible is then given by

$$s^4\cdot 2s^2t^2\cdot 1=2s^6t^2. \tag{2.1}$$

If the second mating is $M\times MN$, then its probability is $2\overline{M}\overline{MN}=4s^3t$ and its true child is M or MN, each being equally probable, among which the case MN alone is distinguishable. Hence, the probability of detection becomes

$$s^4\cdot 4s^3t\cdot \tfrac{1}{2}=2s^7t. \tag{2.2}$$

If the second mating is $N\times N$ then its probability is $\overline{N}^2=t^4$ and its true child is always N. Hence, the probability of detection is

$$s^4\cdot t^4\cdot 1=s^4t^4. \tag{2.3}$$

If the second mating is $N\times MN$, then its probability is $2\overline{N}\overline{MN}=4st^3$ and its true child is N or MN. Hence, the probability of detection is similarly equal to

$$s^4\cdot 4st^3\cdot 1=4s^5t^3. \tag{2.4}$$

If the second mating is $MN\times MN$, then its probability is $\overline{MN}^2=4s^2t^2$ and its true child may be M, N, MN produced at the rate 1/4, 1/4, 1/2, respectively, among which the latter two cases are distinguishable. Hence, the probability of detection is

$$s^4\cdot 4s^2t^2\cdot \tfrac{3}{4}=3s^6t^2. \tag{2.5}$$

Next, we consider the first mating of $M\times N$, its true child being always MN. If the second mating is $M\times MN$, the detection is possible when and only when its child is M, and hence we get the probability

$$2s^2t^2\cdot 4s^3t\cdot \tfrac{1}{2}=4s^5t^3. \tag{2.6}$$

If the second mating is $N\times N$, $N\times MN$ or $MN\times MN$, we get similarly the respective probabilities

$$2s^2t^2\cdot t^4\cdot 1=2s^2t^6, \tag{2.7}$$
$$2s^2t^2\cdot 4st^3\cdot \tfrac{1}{2}=4s^3t^5, \tag{2.8}$$
$$2s^2t^2\cdot 4s^2t^2\cdot \tfrac{1}{2}=4s^4t^4. \tag{2.9}$$

The first mating $M\times MN$ can produce M or MN with equal probability. If the second mating is $N\times N$, the detection is always possible, yielding the probability

$$4s^3t\cdot t^4\cdot 1=4s^3t^5. \tag{2.10}$$

If the second mating is $N\times MN$, then its true child can equally probably be N or MN. Hence, we can detect the interchange except when both children are MN, thus yielding the probability

$$4s^3t\cdot 4st^3\cdot \tfrac{3}{4}=12s^4t^4. \tag{2.11}$$

If the second mating is $MN\times MN$, then its true child can be M, N, or MN among which only the case N with probability 1/4 is detectable. Hence, the probability of detection becomes

$$4s^3t\cdot 4s^2t^2\cdot \tfrac{1}{4}=4s^5t^3. \tag{2.12}$$

The first mating $N\times N$ can produce N alone. If the second mating is $N\times MN$ or $MN\times MN$, then the respective probability of detection is, similarly to (2.2) or (2.5), equal to

$$t^4\cdot 4st^3\cdot \tfrac{1}{2}=2st^7, \tag{2.13}$$
$$t^4\cdot 4s^2t^2\cdot \tfrac{3}{4}=3s^2t^6. \tag{2.14}$$

Last, if the first mating is $N\times MN$ and the second is $MN\times MN$, the detection is possible when and only when the true child of the latter is M. Hence, the probability of detection is, similarly to (2.12), equal to

$$4st^3\cdot 4s^2t^2\cdot \tfrac{1}{4}=4s^3t^5. \tag{2.15}$$

By commuting the roles of the first and the second matings, we get the same results as above. Consequently, *the whole probability of detecting the interchange of infants by means of MN blood type* amounts to the twice of the total sum of (2.1) to (2.15), and hence is equal to

$$\begin{aligned} G_{MN}&=2st(2(s^6+t^6)+5st(s^4+t^4)+12s^2t^2(s^2+t^2)+17s^3t^3)\\ &=2st(2-7st+10s^2t^2-s^3t^3). \end{aligned} \tag{2.16}$$

This is a result coincident with the one previously obtained by Wiener[2]. The above way of derivation has also followed essentially the Wiener's one. The results obtained may be tabulated as follows.

1st mating \ 2nd mating	$M\times M$	$M\times N$	$M\times MN$	$N\times N$	$N\times MN$	$MN\times MN$	Sub-prob. with respect to 1st mating
$M\times M$	0	$2s^6t^2$	$2s^7t$	s^4t^4	$4s^5t^3$	$3s^6t^2$	$s^4t(1+s)$
$M\times N$	$2s^6t^2$	0	$4s^5t^3$	$2s^2t^6$	$4s^3t^5$	$4s^4t^4$	$2s^2t^2(1-2st)$
$M\times MN$	$2s^7t$	$4s^5t^3$	0	$4s^3t^5$	$12s^4t^4$	$4s^5t^3$	$2s^3t(s^4+2t^2(1+s))$
$N\times N$	s^4t^4	$2s^2t^6$	$4s^3t^5$	0	$2st^7$	$3s^2t^6$	$st^4(1+t)$
$N\times MN$	$4s^5t^2$	$4s^3t^5$	$12s^4t^4$	$2st^7$	0	$4s^3t^5$	$2st^3(t^4+2s^2(1+t))$
$MN\times MN$	$3s^6t^2$	$4s^4t^4$	$4s^5t^3$	$3s^2t^6$	$4s^3t^5$	0	$s^2t^2(3-8st+2s^2t^2)$

$$G_{MN}=2st(2-7st+10s^2t^2-s^3t^3)$$

2) A. S. Wiener, Chances of detecting interchange of infants, with special reference to blood groups. Zeitschr. f. indukt. Abstam.-u. Vererbungslehre **59** (1931), 227-235.

—*To be continued*—