136. *Probabilities on Inheritance in Consanguineous Families. X*

By Yûsaku KOMATU and Han NISHIMIYA

Department of Mathematics, Tokyo Institute of Technology

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VIII. Combinations through the most extreme consanguineous marriages

4. Parent-descendants combinations

We shall now deal with *parent-descendants combinations of the extreme mode*. We begin with a combination *immediate after successive consanguineous marriages*, of which the probability is given by

$$\mathfrak{k}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\equiv\kappa_{(11;0)_{t-1}|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$
$$=\sum\kappa(\alpha\beta;\ ab,\ cd)\mathfrak{e}_{t-1}(ab,\ cd;\ \xi_1\eta_1,\ \xi_2\eta_2)$$

or alternatively by

$$\mathfrak{k}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\sum\overline{A}_{ab}\mathfrak{e}_t(ab,\ \alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

Actual computation will lead to the following results:

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ ii)=\frac{1}{2}(1+i)+(1-i)\Big\{-\boldsymbol{R}\frac{7+3\sqrt{5}}{10}\omega^t+\frac{1}{5}\frac{1}{4^t}+i\Big(-\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{2}\frac{1}{2^t}-\frac{3}{10}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ ig)=g\Big\{\boldsymbol{R}\frac{2}{5}\omega^t-\frac{2}{5}\frac{1}{4^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}+\frac{3}{5}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ gg)=g\Big\{\boldsymbol{R}\frac{3-\sqrt{5}}{10}\omega^t-\frac{1}{30}\frac{1}{2^t}+\frac{1}{30}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}-\frac{2}{15}\frac{1}{(-8)^t}+i\Big(\boldsymbol{R}\frac{-2+\sqrt{5}}{10}\omega^t+\frac{1}{30}\frac{1}{2^t}-\frac{2}{15}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)+g\Big(\frac{1}{30}\frac{1}{2^t}+\frac{1}{6}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ii,\ fg)=fg\Big(\frac{1}{15}\frac{1}{2^t}+\frac{1}{3}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ ik)=k\Big\{\boldsymbol{R}\frac{-4+4\sqrt{5}}{5}\omega^t-\frac{4}{15}\frac{1}{2^t}+\frac{2}{15}\frac{1}{4^t}+\frac{2}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}+i\Big(\boldsymbol{R}\frac{6-2\sqrt{5}}{5}\omega^t+\frac{4}{15}\frac{1}{2^t}-\frac{8}{15}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big)+k\Big(\frac{4}{15}\frac{1}{2^t}+\frac{2}{3}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ kk)=k\Big\{\boldsymbol{R}\frac{2}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}\Big)+k\Big(\frac{1}{3}\frac{1}{2^t}-\frac{1}{2}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ ig)=kg\Big(\frac{4}{15}\frac{1}{2^t}+\frac{2}{3}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ kg)=kg\Big(\frac{4}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-4)^t}-\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ ik,\ gg)=kg\Big(\frac{1}{15}\frac{1}{2^t}-\frac{1}{6}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ kk,\ kk)=\frac{1}{2}k+k\Big\{-\boldsymbol{R}\frac{9+4\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}-\frac{1}{10}\frac{1}{4^t}+k\Big(\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{2}\frac{1}{2^t}+\frac{3}{10}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ii;\ kk,kg)=kg\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ kk,gg)=kg\Big(\boldsymbol{R}\frac{-2+\sqrt{5}}{10}\omega^t-\frac{1}{30}\frac{1}{2^t}+\frac{1}{30}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-4)^t}-\frac{2}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ii;\ hk,hk)=hk\Big(\boldsymbol{R}\frac{6-2\sqrt{5}}{5}\omega^t-\frac{4}{15}\frac{1}{2^t}+\frac{2}{15}\frac{1}{4^t}-\frac{4}{3}\frac{1}{(-4)^t}+\frac{4}{15}\frac{1}{(-8)^t}\Big);$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,ii)=\frac{1}{4}(1+2i)+\Big\{-\boldsymbol{R}\frac{9+4\sqrt{5}}{20}\omega^t+\frac{1}{4}\frac{1}{2^t}-\frac{1}{20}\frac{1}{4^t}+i\Big(-\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{2}\frac{1}{2^t}$$
$$+\frac{1}{5}\frac{1}{4^t}\Big)+i^2\Big(\boldsymbol{R}\frac{2+\sqrt{5}}{10}\omega^t-\frac{1}{5}\frac{1}{4^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,ij)=\boldsymbol{R}\frac{-1+\sqrt{5}}{20}\omega^t-\frac{1}{12}\frac{1}{2^t}+\frac{1}{20}\frac{1}{4^t}+\frac{1}{12}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{1}{5}\omega^t+\frac{1}{6}\frac{1}{2^t}-\frac{1}{5}\frac{1}{4^t}$$
$$-\frac{1}{6}\frac{1}{(-4)^t}\Big)+j\Big(\boldsymbol{R}\frac{1}{5}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{20}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}\Big)+i^2\Big(\frac{1}{6}\frac{1}{2^t}+\frac{1}{4}\frac{1}{4^t}$$
$$+\frac{1}{12}\frac{1}{(-4)^t}\Big)+ij\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t+\frac{1}{6}\frac{1}{2^t}-\frac{3}{20}\frac{1}{4^t}+\frac{1}{12}\frac{1}{(-4)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,jj)=\boldsymbol{R}\frac{-2+\sqrt{5}}{20}\omega^t-\frac{1}{60}\frac{1}{2^t}+\frac{1}{60}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{1}{15}\frac{1}{(-8)^t}+(i+j)\Big(\boldsymbol{R}\frac{3-\sqrt{5}}{20}\omega^t$$
$$-\frac{1}{15}\frac{1}{4^t}-\frac{1}{4}\frac{1}{(-4)^t}+\frac{1}{6}\frac{1}{(-8)^t}\Big)+(i^2+j^2)\Big(\frac{1}{60}\frac{1}{2^t}+\frac{1}{12}\frac{1}{(-4)^t}-\frac{1}{10}\frac{1}{(-8)^t}\Big)$$
$$+ij\Big(\boldsymbol{R}\frac{-2+\sqrt{5}}{10}\omega^t+\frac{1}{30}\frac{1}{2^t}+\frac{1}{5}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{1}{5}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,ig)=g\Big\{\boldsymbol{R}\frac{1}{5}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{20}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t+\frac{1}{6}\frac{1}{2^t}-\frac{3}{20}\frac{1}{4^t}$$
$$+\frac{1}{12}\frac{1}{(-4)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,jg)=g\Big\{\frac{1}{30}\frac{1}{2^t}-\frac{1}{12}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}+i\Big(\frac{1}{30}\frac{1}{2^t}+\frac{1}{4}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}$$
$$-\frac{1}{5}\frac{1}{(-8)^t}\Big)+j\Big(\frac{1}{30}\frac{1}{2^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{1}{5}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,gg)=g\Big\{\boldsymbol{R}\frac{3-\sqrt{5}}{20}\omega^t-\frac{1}{30}\frac{1}{2^t}+\frac{1}{60}\frac{1}{4^t}-\frac{1}{6}\frac{1}{(-4)^t}+\frac{1}{30}\frac{1}{(-8)^t}+i\Big(\boldsymbol{R}\frac{-2+\sqrt{5}}{10}\omega^t$$
$$-\frac{1}{20}\frac{1}{4^t}+\frac{1}{4}\frac{1}{(-4)^t}\Big)+g\Big(\frac{1}{60}\frac{1}{2^t}+\frac{1}{12}\frac{1}{(-4)^t}-\frac{1}{10}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ii,fg)=fg\Big(\frac{1}{30}\frac{1}{2^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{1}{5}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ij;\ ij,ij)=\boldsymbol{R}\frac{3-\sqrt{5}}{5}\omega^t-\frac{2}{15}\frac{1}{2^t}+\frac{1}{15}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}+(i+j)\Big(\boldsymbol{R}\frac{-2+2\sqrt{5}}{5}\omega^t$$
$$-\frac{4}{15}\frac{1}{4^t}+\frac{1}{(-4)^t}-\frac{1}{3}\frac{1}{(-8)^t}\Big)+(i^2+j^2)\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{(-4)^t}+\frac{1}{5}\frac{1}{(-8)^t}\Big),$$
$$+ij\Big(\boldsymbol{R}\frac{6-2\sqrt{5}}{5}\omega^t+\frac{4}{15}\frac{1}{2^t}+\frac{4}{5}\frac{1}{4^t}-\frac{2}{3}\frac{1}{(-4)^t}+\frac{2}{5}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{k}_{t-1|11}(ij;\ ij,ig)=g\Big\{\frac{2}{15}\frac{1}{2^t}-\frac{1}{6}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{2}{15}\frac{1}{(-8)^t}+i\Big(\frac{2}{15}\frac{1}{2^t}+\frac{1}{2}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}$$
$$+\frac{1}{5}\frac{1}{(-8)^t}\Big)+g\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{(-4)^t}+\frac{1}{5}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ij,gg)=g\Big\{\frac{1}{30}\frac{1}{2^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{1}{5}\frac{1}{(-8)^t}+(i+j)\Big(\frac{1}{30}\frac{1}{2^t}-\frac{1}{12}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)$$
$$+g\Big(\frac{1}{6}\frac{1}{4^t}+\frac{1}{3}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{k}_{t-1|11}(ij;\ ij,fg)=fg\Big(\frac{1}{3}\frac{1}{4^t}+\frac{2}{3}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ ik)=k\Big\{\boldsymbol{R}\frac{-2+2\sqrt{5}}{5}\omega^t-\frac{4}{15}\frac{1}{2^t}+\frac{1}{15}\frac{1}{4^t}+\frac{2}{3}\frac{1}{(-4)^t}-\frac{1}{15}\frac{1}{(-8)^t}$$
$$+i\Big(\boldsymbol{R}\frac{6-2\sqrt{5}}{5}\omega^t-\frac{1}{5}\frac{1}{4^t}-\frac{1}{(-4)^t}\Big)+k\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{(-4)^t}+\frac{1}{5}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ jk)=k\Big\{\frac{2}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{(-4)^t}+\frac{1}{5}\frac{1}{(-8)^t}+(i+j)\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{6}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big)$$
$$+k\Big(\frac{1}{3}\frac{1}{4^t}-\frac{1}{3}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ kk)=k\Big\{\boldsymbol{R}\frac{1}{5}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{20}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{-1+\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{1}{10}\frac{1}{4^t}$$
$$+\frac{1}{6}\frac{1}{(-4)^t}\Big)+k\Big(\frac{1}{6}\frac{1}{2^t}-\frac{1}{4}\frac{1}{4^t}+\frac{1}{12}\frac{1}{(-8)^t}\Big)\Big\},$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ ig)=kg\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{3}\frac{1}{(-4)^t}+\frac{1}{5}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ jg)=kg\Big(\frac{1}{3}\frac{1}{4^t}-\frac{1}{3}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ kg)=kg\Big(\frac{2}{15}\frac{1}{2^t}-\frac{1}{6}\frac{1}{4^t}+\frac{1}{6}\frac{1}{(-4)^t}-\frac{2}{15}\frac{1}{(-8)^t}\Big),$$

$$\mathfrak{t}_{t-1|11}(ij;\ ik,\ gg)=kg\Big(\frac{1}{30}\frac{1}{2^t}-\frac{1}{12}\frac{1}{4^t}-\frac{1}{12}\frac{1}{(-4)^t}+\frac{2}{15}\frac{1}{(-8)^t}\Big).$$

We next consider a combination in which *one descendant is immediate while another is distant after the last marriage.* It is defined, for $\nu>1$, by an equation

$$\mathfrak{t}_{t-1|1\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\sum\mathfrak{t}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ ab)\kappa_{\nu-1}(ab;\ \xi_2\eta_2)$$

which is brought into the form

$$\mathfrak{t}_{t-1|1\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\mathfrak{t}_{t-1|1}(\alpha\beta;\ \xi_1\eta_1)\overline{A}_{\xi_2\eta_2}+2^{-\nu}W_{t-1}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2).$$

Here the quantity involved in the residual term is defined by

$$W_{t-1}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\sum\mathfrak{t}_{t-1|11}(\alpha\beta;\ \xi_1\eta_1,\ ab)Q(ab;\ \xi_2\eta_2)$$

and its values will be set out in the following lines:

$$W_{t-1}(ii;\ ii,\ ii)=2i(1+i)(1-i)+i(1+i)\Big\{-\boldsymbol{R}\frac{10+6\sqrt{5}}{5}\omega^t+i\Big(\boldsymbol{R}\frac{-5+7\sqrt{5}}{5}\omega^t-\frac{1}{2^t}\Big)$$
$$-i^2\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ii,\ ig)=2g(1+i)(1-2i)+g\Big\{-\boldsymbol{R}\frac{10+6\sqrt{5}}{5}\omega^t+i\Big(\boldsymbol{R}\frac{15+19\sqrt{5}}{5}\omega^t-\frac{1}{2^t}\Big)$$
$$+i^2\Big(\boldsymbol{R}\frac{-20+16\sqrt{5}}{5}\omega^t+2\frac{1}{2^t}\Big)-i^3\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ii,\ gg)=-2g^2(1+i)+g^2\Big\{\boldsymbol{R}\frac{10+6\sqrt{5}}{5}\omega^t+i\Big(-\boldsymbol{R}\frac{-5+7\sqrt{5}}{5}\omega^t+\frac{1}{2^t}\Big)$$
$$-i^2\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ii,\ fg)=-4fg(1+i)+fg\Big\{\boldsymbol{R}\frac{20+12\sqrt{5}}{5}\omega^t+i\Big(-\boldsymbol{R}\frac{-10+14\sqrt{5}}{5}\omega^t+2\frac{1}{2^t}\Big)$$
$$-i^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ ii)=ik\Big\{\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+i\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t+2\frac{1}{2^t}\Big)-i^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ ik)=k\Big\{i\Big(\boldsymbol{R}\frac{8\sqrt{5}}{2}\omega^t-\frac{4}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+k\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+i^2\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$

$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)+ik\Big(-\boldsymbol{R}\frac{-10+34\sqrt{5}}{5}\omega^t+\frac{10}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2k\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ kk)=k^2\Big\{\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$

$$+k\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ ig)=kg\Big\{\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+i\Big(-\boldsymbol{R}\frac{-10+34\sqrt{5}}{5}\omega^t+\frac{10}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$

$$-i^2\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ kg)=kg\Big\{\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$

$$+k\Big(-\boldsymbol{R}\frac{32\sqrt{5}}{5}\omega^t+\frac{8}{3}\frac{1}{2^t}-\frac{8}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ gg)=kg^2\Big\{-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ ik,\ fg)=kfg\Big\{-\boldsymbol{R}\frac{32\sqrt{5}}{5}\omega^t+\frac{8}{3}\frac{1}{2^t}-\frac{8}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ ii)=-2i^2k+ik\Big\{\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{5+9\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}$$

$$+\frac{4}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ ik)=2ik(1-2k)+k\Big\{i\Big(-\boldsymbol{R}\frac{15+7\sqrt{5}}{5}\omega^t+\frac{1}{2^t}\Big)+k\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$

$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)+ik\Big(\boldsymbol{R}\frac{20+16\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+k^2\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)$$

$$-ik^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ kk)=2k^2(1+k)+k^2\Big\{-\boldsymbol{R}\frac{15+7\sqrt{5}}{5}\omega^t+\frac{1}{2^t}+k\Big(\boldsymbol{R}\frac{15+7\sqrt{5}}{5}\omega^t-\frac{1}{2^t}\Big)$$

$$-k^2\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ ig)=-4ikg+kg\Big\{\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10+18\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$

$$+\frac{8}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ kg)=2kg(1-2k)+kg\Big\{-\boldsymbol{R}\frac{15+7\sqrt{5}}{5}\omega^t+\frac{1}{2^t}+k\Big(\boldsymbol{R}\frac{20+16\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}$$

$$+\frac{4}{3}\frac{1}{(-4)^t}\Big)-k^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ gg)=-2kg^2-kg^2\Big\{\boldsymbol{R}\frac{5+9\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}-k\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ kk,\ fg)=-4kfg-kfg\Big\{\boldsymbol{R}\frac{10+18\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{8}{3}\frac{1}{(-4)^t}-k\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ ii)=ihk\Big\{\frac{4}{3}\frac{1}{2^t}+\frac{8}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ ik)=hk\Big\{\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}+k\Big(\frac{4}{3}\frac{1}{2^t}+\frac{8}{3}\frac{1}{(-4)^t}\Big)$$

$$-ik\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ kk)=hk^2\Big\{\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}-k\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ hk)=hk\Big\{(h+k)\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-hk\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ ig)=hkg\Big\{\frac{4}{3}\frac{1}{2^t}+\frac{8}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ kg)=hkg\Big\{\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}-k\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ii;\ hk,\ gg)=-hkg^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t,$$

$$W_{t-1}(ii;\ hk,\ fg)=-hkfg\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t;$$

$$W_{t-1}(ij;\ ii,\ ii)=i(1-i)(1+2i)+i\Big\{-\boldsymbol{R}\frac{15+7\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}+i\Big(\boldsymbol{R}\frac{2\sqrt{5}}{5}\omega^t-\frac{1}{2^t}\Big)$$
$$+i^2\boldsymbol{R}\frac{15-\sqrt{5}}{5}\omega^t-i^3\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ii,\ ij)=j(1-2i)(1+2i)+i\Big(\boldsymbol{R}\frac{-5+3\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+j\Big(-\boldsymbol{R}\frac{15+7\sqrt{5}}{10}\omega^t$$
$$+\frac{1}{2}\frac{1}{2^t}\Big)+i^2\Big(\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t+\frac{1}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+ij\Big(\boldsymbol{R}\frac{10+4\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$-i^2\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2j\Big(\boldsymbol{R}4\omega^t+\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-i^3j\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t,$$

$$W_{t-1}(ij;\ ii,\ jj)=-j^2(1+2i)+j\Big\{\boldsymbol{R}\frac{-5+3\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t$$
$$+\frac{1}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+j\Big(\boldsymbol{R}\frac{10+2\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$+ij\Big(\boldsymbol{R}\frac{5+\sqrt{5}}{5}\omega^t+\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-i^2j\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ii,\ ig)=g(1-2i)(1+2i)+g\Big\{-\boldsymbol{R}\frac{15+7\sqrt{5}}{10}\omega^t+\frac{1}{2}\frac{1}{2^t}+i\Big(\boldsymbol{R}\frac{10+4\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}$$
$$-\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(\boldsymbol{R}4\omega^t+\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-i^3\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ii,\ jg)=-2jg(1+2i)+g\Big\{\boldsymbol{R}\frac{-5+3\sqrt{5}}{10}\omega^t-\frac{1}{6}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t$$
$$+\frac{1}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+j\Big(\boldsymbol{R}\frac{20+4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$+ij\Big(\boldsymbol{R}\frac{10+2\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2j\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)\Big\},$$

$$W_{t-1}(ij;\ ii,\ gg)=-g^2(1+2i)+g^2\Big\{\boldsymbol{R}\frac{10+2\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{5+\sqrt{5}}{5}\omega^t$$
$$+\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-i^2\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ii,\ fg)=-2fg(1+2i)+fg\Big\{\boldsymbol{R}\frac{20+4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10+2\sqrt{5}}{5}\omega^t$$
$$+\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ij,\ ii)=i\Big\{\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{-10+6\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$+j\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(-\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$+ij\Big(\boldsymbol{R}(2-2\sqrt{5})\omega^t+\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)-i^2j\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ij,\ ij)=(i+j)\Big(\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)+(i^2+j^2)\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$

$$+\frac{2}{3}\frac{1}{(-4)^t}\Big)+ij\Big(\boldsymbol{R}\frac{-20+12\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{8}{3}\frac{1}{(-4)^t}\Big)+ij(i+j)\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t$$
$$+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-2i^2j^2\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t,$$

$$W_{t-1}(ij;\ ij,\ ig)=g\Big\{\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(-\boldsymbol{R}\frac{20-8\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}+\frac{8}{3}\frac{1}{(-4)^t}\Big)$$
$$+j\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$+ij\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2j\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ij,\ gg)=g^2\Big\{-\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}+(i+j)\Big(-\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}$$
$$-\frac{2}{3}\frac{1}{(-4)^t}\Big)-ij\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ij,\ fg)=fg\Big\{-\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2}+\frac{8}{3}\frac{1}{(-4)^t}+(i+j)\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}$$
$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)-ij\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ ii)=ik\Big\{\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}(2-2\sqrt{5})\omega^t+\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$-i^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ ij)=k\Big\{i\Big(\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)+j\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(\frac{2}{3}\frac{1}{2^t}$$
$$+\frac{4}{3}\frac{1}{(-4)^t}\Big)+ij\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2j\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ jj)=jk\Big\{\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+j\Big(-\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}$$
$$-\frac{2}{3}\frac{1}{(-4)^t}\Big)-ij\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ ik)=k\Big\{(i+k)\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+i^2\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$
$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)+ik\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2k\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ jk)=k\Big\{j\Big(\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$+ij\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+ik\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$+jk\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-ijk\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ kk)=k^2\Big\{\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$+k\Big(-\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ ig)=kg\Big\{\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(-\boldsymbol{R}\frac{-10+18\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}$$
$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)-i^2\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ jg)=kg\Big\{\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+j\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}$$
$$-\frac{4}{3}\frac{1}{(-4)^t}\Big)-ij\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ kg)=kg\Big\{\boldsymbol{R}\frac{4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$+k\Big(-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ gg)=kg^2\Big\{-\boldsymbol{R}\frac{8\sqrt{5}}{5}\omega^t+\frac{2}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ ik,\ fg)=kfg\Big\{-\boldsymbol{R}\frac{16\sqrt{5}}{5}\omega^t+\frac{4}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ kk,\ ii)=-2i^2k+ik\Big\{\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{5+9\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}$$
$$+\frac{4}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ kk,\ ij)=-4ijk+k\Big\{(i+j)\Big(\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)+ij\Big(\boldsymbol{R}\frac{10+18\sqrt{5}}{5}\omega^t$$
$$-\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+(i+j)k\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-ijk\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ kk,\ ik)=2ik(1-2k)+k\Big\{i\Big(-\boldsymbol{R}\frac{15+7\sqrt{5}}{5}\omega^t+\frac{1}{2^t}\Big)+k\Big(\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}\Big)$$
$$+ik\Big(\boldsymbol{R}\frac{20+16\sqrt{5}}{5}\omega^t-\frac{4}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)+k^2\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-ik^2\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ kk,\ ig)=-4ikg+kg\Big\{\boldsymbol{R}\frac{5-\sqrt{5}}{5}\omega^t-\frac{1}{3}\frac{1}{2^t}-\frac{2}{3}\frac{1}{(-4)^t}+i\Big(\boldsymbol{R}\frac{10+18\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}$$
$$+\frac{8}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{1}{3}\frac{1}{2^t}+\frac{2}{3}\frac{1}{(-4)^t}\Big)-ik\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ hk,\ ii)=ihk\Big\{\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{20-4\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ hk,\ ij)=hk\Big\{(i+j)\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)-ij\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ hk,\ ik)=hk\Big\{i\Big(\boldsymbol{R}\frac{10-2\sqrt{5}}{5}\omega^t-\frac{2}{3}\frac{1}{2^t}-\frac{4}{3}\frac{1}{(-4)^t}\Big)+k\Big(\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}\Big)$$
$$-ik\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\},$$

$$W_{t-1}(ij;\ hk,\ ig)=hkg\Big\{\frac{2}{3}\frac{1}{2^t}+\frac{4}{3}\frac{1}{(-4)^t}-i\boldsymbol{R}\frac{40-8\sqrt{5}}{5}\omega^t\Big\}.$$

We finally consider a combination in which *both descendants are distant after the last marriage.* Its defining equation is given, for $\mu,\ \nu>1$, in the form

$$\mathfrak{k}_{t-1|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\equiv\kappa_{(11;0)t-1|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)$$
$$=\sum\mathfrak{k}_{t-1|1\nu}(\alpha\beta;\ ab,\ \xi_2\eta_2)\kappa_{\mu-1}(ab;\ \xi_1\eta_1),$$

whence follows a desired formula

$$\mathfrak{k}_{t-1|\mu\nu}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=\overline{A}_{\xi_1\eta_1}\overline{A}_{\xi_2\eta_2}+2^{-\mu+1}\overline{A}_{\xi_2\eta_2}Q(\alpha\beta;\ \xi_1\eta_1)$$
$$+2^{-\nu+1}A_{\xi_1\eta_1}Q(\alpha\beta;\ \xi_2\eta_2)+2^{-\lambda}T_{t-1}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)\qquad(\lambda=\mu+\nu-1).$$

The values of the quantity

$$T_{t-1}(\alpha\beta;\ \xi_1\eta_1,\ \xi_2\eta_2)=2\sum W_{t-1}(\alpha\beta;\ ab,\ \xi_2\eta_2)Q(ab;\ \xi_1\eta_1),$$

symmetric with respect to $\xi_1\eta_1$ and $\xi_2\eta_2$, will be set out in the following lines:

$$T_{t-1}(ii;\ ii,\ ii)=4i^2(1-i)+4i^2(1-i)\boldsymbol{R}\Big(-\frac{5+\sqrt{5}}{5}-i\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ii,\ ig)=4ig(1-2i)+4ig\boldsymbol{R}\Big(-\frac{5+\sqrt{5}}{5}+i\frac{10+\sqrt{5}}{5}+i^2\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ii,\ gg)=-4ig^2+4ig^2\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ii,\ fg)=-8ifg+8ifg\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ ik)=4k(k+i^2-3ik)+4k\boldsymbol{R}\Big(-k\frac{5+\sqrt{5}}{5}+ik\frac{15+\sqrt{5}}{5}+i^2\frac{5+\sqrt{5}}{5}+i^2k\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ kk)=4k^2(i-k)+4k^2\boldsymbol{R}\Big(-i\frac{5+2\sqrt{5}}{5}+k\frac{5+\sqrt{5}}{5}+ik\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ ig)=2kg(1-3i)+4kg\boldsymbol{R}\Big(-\frac{5+\sqrt{5}}{5}+i\frac{15+2\sqrt{5}}{5}+i^2\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ kg)=4kg(i-2k)+4kg\boldsymbol{R}\Big(-i\frac{5+2\sqrt{5}}{10}+k\frac{10+2\sqrt{5}}{5}+ik\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ gg)=-4kg^2+8kg^2\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ ik,\ fg)=-8kfg+16kfg\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ kk,\ kk)=4k^3+4k^2\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}+k\frac{\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ kk,\ ig)=-4k^2g+4k^2g\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ kk,\ kg)=4k^2g+4k^2g\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}+k\frac{2\sqrt{5}}{2}\Big)\omega^t,$$

$$T_{t-1}(ii;\ kk,\ gg)=4k^2g^2\boldsymbol{R}\frac{\sqrt{5}}{5}\omega^t,$$

$$T_{t-1}(ii;\ kk,\ fg)=8k^2fg\boldsymbol{R}\frac{\sqrt{5}}{5}\omega^t,$$

$$T_{t-1}(ii;\ hk,\ hk)=4hk(h+k)+4hk\boldsymbol{R}\Big(-(h+k)\frac{5+2\sqrt{5}}{5}+hk\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ hk,\ kg)=4hkg+4hkg\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}+k\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ii;\ hk,\ fg)=16hkfg\boldsymbol{R}\frac{\sqrt{5}}{5}\omega^t;$$

$$T_{t-1}(ij;\ ii,\ ii)=2i^2+2i^2\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}-i\frac{2\sqrt{5}}{5}+i^2\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ ij)=2i(j-i^2-ij)+2i\boldsymbol{R}\Big(i\frac{\sqrt{5}}{5}-j\frac{5+2\sqrt{5}}{5}+i^2\frac{5+\sqrt{5}}{5}+ij\frac{5-\sqrt{5}}{5}+i^2j\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ jj)=-2ij(i+j)+2ij\boldsymbol{R}\Big(\frac{\sqrt{5}}{5}+(i+j)\frac{5+\sqrt{5}}{5}+ij\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ ig)=2ig(1-i)+2ig\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}+i\frac{5-\sqrt{5}}{5}+i^2\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ jg)=-2ig(i+2j)+2ig\boldsymbol{R}\Big(\frac{\sqrt{5}}{5}+i\frac{5+\sqrt{5}}{5}+j\frac{10+2\sqrt{5}}{5}+ij\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ gg)=-2ig^2+2ig^2\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ii,\ fg)=-4ifg+4ifg\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{2\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ij,\ ij)=2(i^2+j^2-2ij(i+j))+2\boldsymbol{R}\Big(-(i^2+j^2)\frac{5+2\sqrt{5}}{5}+ij\frac{2\sqrt{5}}{5}+ij(i+j)2$$
$$+i^2j^2\frac{8\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ij,\ ig)=2g(j-2i^2-2ij)+2g\boldsymbol{R}\Big(i\frac{\sqrt{5}}{5}-j\frac{5+2\sqrt{5}}{5}+i^2\frac{10+2\sqrt{5}}{5}+ij2+i^2j^2\frac{8\sqrt{5}}{5}\Big)\omega^t$$

$$T_{t-1}(ij;\ ij,\ gg)=-2g^2(i+j)+2g^2\boldsymbol{R}\Big((i+j)\frac{5+\sqrt{5}}{5}+ij\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ij, fg)=-4fg(i+j)+4fg\,\boldsymbol{R}\Big((i+j)\frac{5+\sqrt{5}}{5}+ij\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ ik)=2k(k+2i^2-2ik)+2k\boldsymbol{R}\Big(-k\frac{5+2\sqrt{5}}{5}+i^2\frac{10+4\sqrt{5}}{5}+ik2+i^2k\frac{8\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ jk)=4k(ij-(i+j)k)+2k\boldsymbol{R}\Big(k\frac{\sqrt{5}}{5}-ij\frac{10+4\sqrt{5}}{5}+(i+j)k\frac{10+2\sqrt{5}}{5}+ijk\frac{8\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ kk)=2k^2(2i-k)+2k^2\boldsymbol{R}\Big(-i\frac{10+4\sqrt{5}}{5}+k\frac{5+\sqrt{5}}{5}+ik\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ ig)=2kg(1-2i)+2kg\boldsymbol{R}\Big(-\frac{5+2\sqrt{5}}{5}+i2+i^2\frac{8\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ jg)=-4kg(i+j)+2kg\boldsymbol{R}\Big(\frac{\sqrt{5}}{5}+(i+j)\frac{10+2\sqrt{5}}{5}+ij\frac{8\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ kg)=4kg(i-k)+4kg\,\boldsymbol{R}\Big(-i\frac{5+2\sqrt{5}}{5}+k\frac{5+\sqrt{5}}{5}+ik\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik,\ gg)=-2kg^2+2kg^2\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{4\sqrt{5}}{5}\Big)\omega^t,$$

$$T_{t-1}(ij;\ ik, fg)=-4kfg+4kfg\boldsymbol{R}\Big(\frac{5+\sqrt{5}}{5}+i\frac{4\sqrt{5}}{5}\Big)\omega^t.$$

It would be noted that the results derived in the preceding section can be obtained again from those in the present section by eliminating a type of one descendant. Namely, there holds, for any $\nu\geqq 1$, an identity

$$\mathfrak{k}_{t-1|n}(\alpha\beta;\ \xi\eta)=\sum \mathfrak{k}_{t-1|n\nu}(\alpha\beta;\ \xi\eta,\ ab).$$