88. A Remark concerning Probabilities on Inheritance in Consanguineous Families

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In a series of papers[1] we have systematically derived formulas for probabilities of various combinations of consanguineous genotypes.

Having commenced with simple lineal combination, we first have shown that the reduced probability of mother-nth descendant combination is given by[2]

$$\kappa_n(\alpha\beta;\ \xi\eta)=\bar{A}_{\xi\eta}+2^{-n+1}Q(\alpha\beta;\ \xi\eta),$$

and then that the reduced probability of parents-nth descendant combination is given by[3]

$$\varepsilon_n(\alpha\beta,\ \gamma\delta;\ \xi\eta)=\bar{A}_{\xi\eta}+2^{-n+1}E(\alpha\beta,\ \gamma\delta;\ \xi\eta).$$

The values of the quantities Q and E are also fully set out there.

Comparison of these listed tables shows now that there holds a very remarkable relation

$$E(\alpha\beta,\ \gamma\delta;\ \xi\eta)=Q(\alpha\beta;\ \xi\eta)+Q(\gamma\delta;\ \xi\eta).$$

It is to be regretted that we have overlooked this important fact. If it had been perceived at that time, several relations concerning E would have been much simply established.

Especially, the notation $C(\alpha\beta,\ \gamma\delta;\ \xi\eta)$ introduced[4] for expressing the probability of ancestor-parent-descendant combination would then become superfluous, since it is dependent. In fact, we have

$$\begin{aligned} C(\alpha\beta,\ \gamma\delta;\ \xi\eta) &= 2\sum_{a\leqq b} C_0(\alpha\beta,\ \gamma\delta;\ ab)\kappa(ab;\ \xi\eta) \\ &= 2\sum_{a\leqq b} C_0(\alpha\beta,\ \gamma\delta;\ ab)Q(ab;\ \xi\eta) \\ &= 4\sum_{c\leqq d;\, a\leqq b} Q(\alpha\beta;\ cd)\varepsilon(cd,\ \gamma\delta;\ ab)Q(ab;\ \xi\eta) \\ &= 2\sum_{c\leqq d} Q(\alpha\beta;\ cd)E(cd,\ \gamma\delta;\ \xi\eta) \\ &= 2\sum_{c\leqq d} Q(\alpha\beta;\ cd)\{Q(cd;\ \xi\eta)+Q(\gamma\delta;\ \xi\eta)\} \\ &= Q(\alpha\beta;\ \xi\eta). \end{aligned}$$

Thus, for the probability of ancestors-descendant combination

1) Y. Komatu and H. Nishimiya, Probabilities on inheritance in consanguineous families. I-XIII. Proc. Japan Acad. **30** (1954), 42-45; 46-48; 49-52; 148-151; 152-155; 236-240; 241-244; 245-247; 636-640; 641-649; 650-654; **31** (1955), 186-189; 190-194.

2) Cf. I, p. 43.

3) Cf. I, p. 44.

4) Cf. IV, p. 149.

$\varepsilon_{\mu\nu;n}(\alpha\beta, \gamma\delta; \xi\eta)$, the distinction in the formulas according to $\mu=\nu=0$, $\mu>0=\nu$ (or $\mu=0<\nu$), and $\mu, \nu>0$ is really unnecessary provided $n>1$. Three formulas[5] may be unified, in general, into a single formula

$$\varepsilon_{\mu\nu;n}(\alpha\beta, \gamma\delta; \xi\eta)=\overrightarrow{A}_{\xi\eta}+2^{-n+1}\{2^{-\mu}Q(\alpha\beta; \xi\eta)+2^{-\nu}Q(\gamma\delta; \xi\eta)\}$$

valid for any $\mu, \nu \geqq 0$ provided $n>1$.

Based on the identity just mentioned, several relations would then be accordingly more readily deducible.

5) Cf. I, p. 44; IV, p. 149; IV, p. 150.