102. Note on Algebras of Bounded Representation Type

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1. Let A be an associative algebra with a unit element over a field k, N be the radical of A and $g_A(d)$ be the number of inequivalent indecomposable representations of A of degree d where d is an integer. Now A is said to be of bounded representation type if there exists an integer d_0 such that $g_A(d)=0$ for all $d \ge d_0$ and A is said to be of finite representation type if $\sum_{i} g_A(d)$ is finite.¹⁾

Concerning these classes of algebras of bounded type and finite type, Professor Brauer and Professor Thrall conjectured that these two classes are identical.²⁾ This conjecture is not yet proved, but now we shall prove it in a special case where $N^2=0$ and k is algebraically closed.

2. By the same way as [2] we may assume that A is the basic algebra. Then for this purpose we have only to prove that arbitrary two representations by indecomposable A-left modules $\mathfrak{M}_1 = \sum_{i=1}^r \sum_{\lambda_i=1}^{s_i} Ae_i m_{i,\lambda_i}$ and $\mathfrak{M}_2 = \sum_{i=1}^r \sum_{\lambda_i=1}^{s_i} Ae_i m'_{i,\lambda_i}$ which have the same type are equivalent. For the number of such A-left modules with different types is finite.³⁾

Now from the proof of the main theorem of [2], we may consider about the following three cases:

(a) $\{Ne_1, \dots, Ne_r\}$ is such a chain that Ne_r is the direct sum of three simple components.

(b) $\{Ne_1, Ne_2, Ne_3\}$ is such a chain that Ne_2 is the direct sum of three simple components.

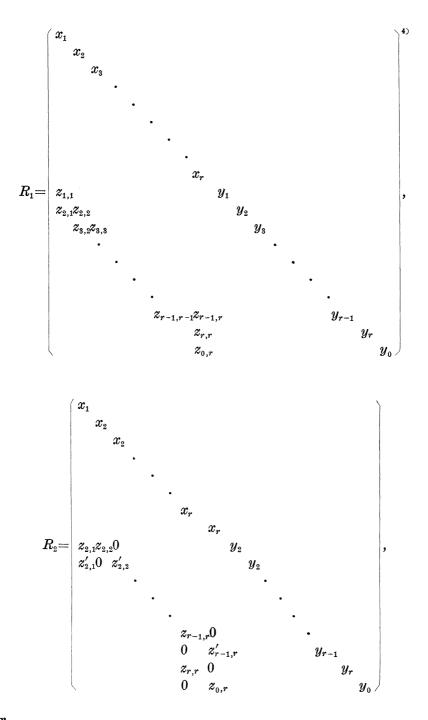
(c) $\{Ne_1, Ne_2, Ne_3, Ne_4\}$ is such a chain that Ne_2 is the direct sum of three simple components and Ne_1 and Ne_4 are simple.

(i) First suppose that $\{Ne_i, \dots, Ne_r\}$ is such a chain as (a). Then an arbitrary indecomposable representation by $\mathfrak{M} = \sum_i \sum_{\lambda_i} Ae_i m_{i,\lambda_i}$ has the following form:

¹⁾ James P. Jans [1].

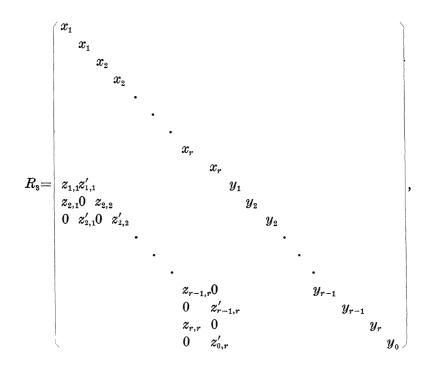
²⁾ James P. Jans [1].

³⁾ In this paper we use the results of [2] without proof.

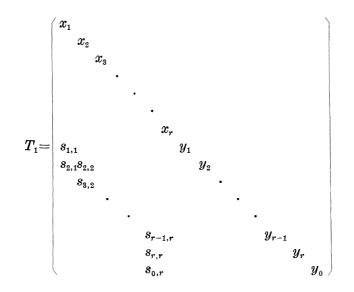


 \mathbf{or}

4) The empty place means zero.

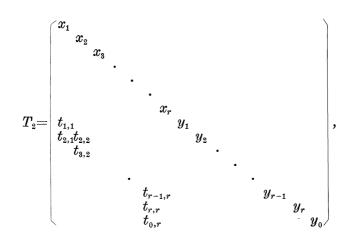


and it is shown by the following way that there exists non singular matrix P such that $PT_1 = T_2P$ for arbitrary two indecomposable representations T_1, T_2 which have the same form as R_1 , namely



and

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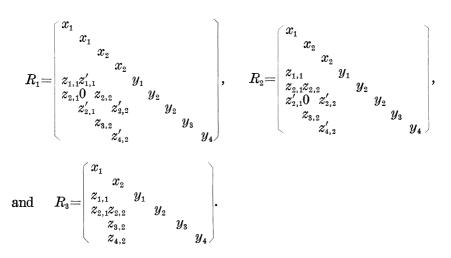
where $t_{i,j} \neq s_{i,j}$.

Now if we put
$$P=(p_{i,j})$$
, we have $p_{i,j}=0$ for $i \leq j$ and
 $t_{1,1}p_{1,1}=p_{r+1,r+1}s_{1,1}, \quad t'_{2,1}p_{1,1}=p_{r+2,r+2}s'_{2,1},$
 $t_{2,2}p_{2,2}=p_{r+2,r+2}s_{2,1}, \quad t_{3,2}p_{2,2}=p_{r+3,r+3}s_{3,2},$
 $\dots \dots \dots$
 $t_{r,r}p_{r,r}=p_{2r,2r}s_{r,r}, \quad t_{0,r}p_{r,r}=p_{2r+1,2r+1}s_{0,r}.$

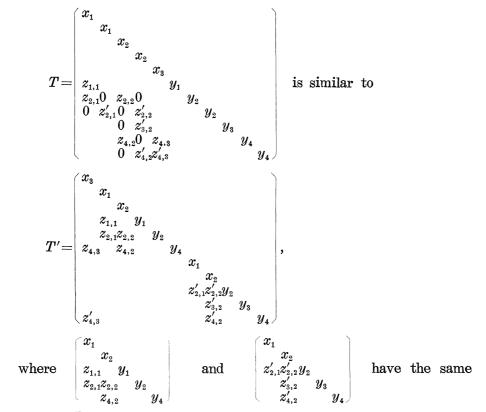
Thus there exists P such that $|P| \neq 0$.

By the same way as this computation we can prove it in the case where T_i has the same form as R_1 or R_2 .

(ii) Now suppose that $\{Ne_1, Ne_2, Ne_3\}$ is such a chain as (b). Then from the result of [2] an arbitrary indecomposable representation is composed of representations of the following types:



For example,



type as R_3 .

Then it is shown by the same way as (i) that there exists non singular matrix P such that $PT_1 = T'_1 P$ where T_1 has the same form as T and T'_1 has $u_{i,j} \neq z_{i,j}$ in place of $z_{i,j}$ of T_1 .

In the case where T_1 has other form given in [2], this is proved by the same way as above.

(iii) Suppose that $\{Ne_1, Ne_2, Ne_3, Ne_4\}$ is such a chain as (c). Then this is proved by the same way as above.

Thus we have the following

Theorem. Let $N^2=0$ and k be algebraically closed. Then the class of algebras of bounded representation type and that of algebras of finite representation type are identical.

References

- [1] James P. Jans: On the indecomposable representations of algebras, Dissertation, University of Michigan (1954).
- [2] T. Yoshii: On algebras of bounded representation type, Osaka Math. Jour., 8, No. 1 (1956).