57. *Note on the Existence of Rational Points*

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H. Nishimura proved the following theorem on the existence of rational points on algebraic varieties:[1]

Let U and V be varieties defined over k and let V be complete and let φ be a rational mapping from U into V. If there exists a k-rational simple point P on U, then there exists a k-rational point on V which corresponds to P by φ.

In the present note a simplification and a generalization of the theorem will be given.

First we remark a property of regular local rings which might be well known.

Lemma. Let R be a regular local ring with dimension r and L the quotient field of R. Then there exists a valuation of L, whose valuation-ring and -ideal contains the ring R and the maximal ideal of R respectively, and whose residue field is isomorphic, in a natural way, with the residue field of R with respect to the maximal ideal of R.

Proof. We shall prove the lemma by induction on the dimension of R. When R is of dimension 1, R is a valuation ring and R itself is the required ring. Consider the case when R is of dimension r. Assuming that the assertion is true for lower dimensional case, let f be an element of the maximal ideal m of R not in m^2, in other words, f is a member of a regular system of parameters. Then fR is a prime ideal of rank 1 and so the quotient ring of R with respect to fR is a valuation ring. We shall denote this valuation by v. The residue ring R/fR is a regular local ring of dimension $r-1$ and its quotient field L' is the residue field of v. By our induction assumption, there exists a valuation $\bar{v}$ of L' such that the residue field of $\bar{v}$ is isomorphic to that of R/fR (hence, to that of R). Then the composite v' of the valuation v and $\bar{v}$ has the required properties.

Theorem.[2] Let U and V be models of function fields L and L' respectively over a ground ring I. Assume that V is complete and that L contains L'. If a spot $P \in U$ is a simple spot, then there exists

1) Some remark on rational points, Mem. Coll. Sci. Univ. Kyoto, **29**, 189-192 (1955).

2) We use the terminologies in M. Nagata's paper: A general theory of algebraic geometry over Dedekind domains I, Amer. Jour. Math., **78**, 78-116 (1956).

a spot $Q \in V$ which corresponds to P such that the residue field of Q is contained in that of P.

Proof. By the above lemma, there exists a place v which dominates P such that the residue field of v coincides with that of P. Since V is complete, v has a center Q on V, which has obviously the required property.

If we translate the above theorem in terms of usual algebraic geometry over a ground field k, then we can express it as follows:

Let U and V be abstract varieties over the same ground field k and let V be complete. And let P be a relatively simple point of U over k. If there exists a rational mapping from U into V defined over k. Then there exists at least one point on V corresponding to P such that $K(Q)$ is contained in $K(P)$.

As a special case of this statement, we have the Nishimura's theorem.