60. *Unsolved Problems of Number Theory*

Steinhaus Problem

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The object of the present note is to state some unsloved problems theoretically related to the Steinhaus problem. The exact formulation of the Steinhaus problem is due to K. Iséki [1] and [2]. By using electronic computers, some numerical calculations of the cyclic parts of it was studied by T. Kusakabe, K. Shibamura, some of our colleagues, and us. For the details of cyclic parts of power 4 and 5, see our papers [3], [4] to be appeared in the Acta Arithmetica.

Now we shall state some questions:

1) *Find the function $f(k)$ that gives the number of distinct cyclic parts for every power k.*

We knew $f(2)=2$, $f(3)=9$, $f(4)=6$ and $f(5)=15$. We do not know the concrete representation of $f(k)$.

2) *Find the length $l(k)$ of cyclic parts for any power k. Can we decide the largest length $L(k)$ of cyclic parts for any power k?* For $L(k)$, we knew

$$L(2)=8,\ L(3)=3,\ L(4)=7 \text{ and } L(5)=28.$$

3) *Is there a cyclic part with the length 1 for every power $k(\geq 3)$ except the trivial case?*

Such a part does not exist for $k=2$. For $k=3, 4, 5$, there are non trivial cyclic parts with the length 1. As a related problem, are there non trivial cyclic parts with length 1 for infinitely many k?

4) If the solution of problem 3) is negative, *find the smallest length of cyclic parts for any power k.*

5) *For a given natural number n, is there the power k having a cyclic part with the length n?*

For $n=2$, 3, 4, 6, 7, 10, 12, 22, and 28, we can find the powers.

Theoretically these problems seem to be very difficult to solve, and have no relations with many well known problems on the number theory.

References

[1] K. Iséki: A problem of number theory, Proc. Japan Acad., **36**, 578-583 (1960).
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[3] K. Chikawa, K. Iséki, and T. Kusakabe: On a problem by H. Stienhaus, Acta Arithmetica, **7** (1961) to be appeared soon.
[4] K. Chikawa, K. Iséki, T. Kusakabe, and K. Shibamura: Computation of cyclic parts of Steinhaus problem for power 5, Acta Arithmetica, **7** (1961) to be appeared soon.