170. On a Theorem of Wielandt

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Apart from the alternating and symmetric groups, there are only four groups known which are quadruply transitive. These are the Mathieu groups M_{11}, M_{12}, M_{23} and M_{24} on 11, 12, 23 and 24 letters, respectively, of which M_{12} and M_{24} are quintuply transitive.

Concerning the existence of multiply transitive groups other than the alternating and symmetric groups, H. Wielandt [2] obtained an interesting result. The theorem of Wielandt is as follows:

Let G be an 8-fold transitive groups of degree n. If the outer automorphism group of any simple subgroup of G is solvable, then G is S_n or A_n.

Improving the theorem of Wielandt we have

Theorem. *Let G be a 7-fold transitive group of degree n satisfying the same assumption as above. Then G is S_n or A_n.*

This theorem is obtained immediately from a lemma (2) in [2] and the following

Proposition. *Let G be a quintuply transitive group on* $\{1, 2, \cdots, n\}$ and *H be the subgroup of G consisting of all the elements leaving the three letters* 1, 2 *and* 3 *invariant. If H contains a normal subgroup Q which is regular on* $\{4, 5, \cdots, n\}$, *then G is one of the following groups*: S_5, S_6, S_7, A_7 *or* M_{12}.

Under the assumption of the proposition, by using a theorem of Jordan ([1], p. 72), we can show that Q is an elementary abelian group of exponent 2 or 3. When the exponent is 3, we can prove that n must be 6, 12 or 30. The case of $n=30$ will be excluded by a theorem of Miller ([1], Theorem 5.7.2). For $n=6$ or 12, G is S_6 or M_{12}.

When the exponent is 2, we can say more. Namely we have

Proposition. *Let G be a quadruply transitive group on* $\{1, 2, \cdots, n\}$ *and H be the subgroup of G consisting of all the elements leaving the two letters* 1 *and* 2 *invariant. If n is even and H contains a normal subgroup which is regular on* $\{3, 4, \cdots, n\}$, *then G is one of the following groups*: S_4, S_6 *or* A_6.

The detailed proofs of the propositions will be given elsewhere.

References

[1] M. Hall: The Theory of Groups. Macmillan, New York (1959).
[2] H. Wielandt: Über den Transitivitätsgrad von Permutationsgruppen. Math. Zeitschr., **74**, 297-298 (1960).