143. On Axiom Systems of Propositional Calculi. V

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In the first note of this series, Y. Imai and K. Iséki proved that Hilbert axioms of two valued propositional calculus imply (F), (R), (L_1), (L_2), (L_3), (S_1), and (S_2) axioms systems (For notations and rules of inference, see [3]). In the second note [2], Y. Arai proved that the (L_1)-system is equivalent to a modification of (L_1)-system. In our seminar, he showed that axioms of the system are independent each other. In a later paper, we shall publish the result with other results on independences of axioms. In the third note (2), Y. Arai published deductions from the (L_3)-system to all other axiom systems. In his note [4], K. Iséki proved that an axiom system given by E. Mendelson implies all other systems of axioms mentioned in [3].

In this note, we shall prove that the Russell axioms of propositional calculus imply Lukasiewicz (L_1)-system. In the next note, S. Tanaka, one of the present authors, proves that (R)-system implies all other systems of axioms of propositional calculus given in [3] and [4]. As already said, we use only two rules of inference, i.e. rules of substitution and detachment.

1 *CpCqp*,
2 *CCpqCCqrCpr*,
3 *CCpCqrCqCpr*,
4 *CNNpp*,
5 *CCpNpNp*,
6 *CCpNqCqNp*.

These six theses are an axiom system given by B. Russell. We shall prove the following theses from these axioms:

3 *r*/*p* **C*1—7,

7 *CqCpp*.

7 *q*/*CpCqp* **C*1—8,

8 *Cpp*.

6 *p*/*Np*, *q*/*p* **C*8 *p*/*Np*—9,

9 *CpNNp*.

3 *p*/*Cpq*, *q*/*Cqr*, *r*/*Cpr* **C*2—10,

10 *CCqrCCpqCpr*.

10 *p*/*q*, *q*/*NNp*, *r*/*p* **C*4—11,

11 *CCqNNpCqp*.

2 *p*/*CNpNq*, *q*/*CqNNp*, *r*/*Cqp* **C*6 *p*/*Np*—*C*11—12,

12 *CCNpNqCqp*.

2 p/Np, q/CNqNp, r/Cpq *C1 p/Np, q/Nq—C12 p/q, q/p—13,

13 CNpCpq.

3 p/Np, q/p, r/p *C13—14,

14 CpCNpq.

10 r/NNq *C4—15,

15 CCpqCpNNq.

2 p/Cpq, q/CpNNq, r/CNqNp *C15—C6 q/Nq—16

16 CCpqCNqNp.

10 q/NNq, r/q *C4 p/q—17,

17 CCpNNqCpq.

10 p/Np, r/NNq *C9 p/q—18,

18 CCNpqCNpNNq.

2 p/CNpq, q/CNpNNq, r/CNqp *C18—C12 q/Nq—19,

19 CCNpqCNqp.

2 p/NNp, q/p, r/p *C4—20,

20 CCpqCNNpq.

2 p/Cpq, q/CNNpq *C20—21,

21 CCCNNpqrCCpqr.

2 p/Cpq, q/CCqrCpr, r/s *C2—22,

22 CCCCqrCprsCCpqs.

22 s/CCCprsCCqrs *C2 p/Cqr, q/Cpr, r/s—23,

23 CCpqCCCprsCCqrs.

3 p/Cpq, q/CCprs, r/CCqrs *C23—24,

24 CCCprsCCpqCCqrs.

24 p/CNNpq, s/CCpqr, q/CNqNp, *C21—C19—25,

25 CCCNqNprCCpqr.

25 q/p, p/Np, r/NNp *C5 p/Np—26,

26 CCNppNNp.

17 p/CNpp, q/p *C26—27,

27 CCNppp.

Therefore, we have the axioms of (L_1)-systems i.e. theses 2, 14, and 27.

References

[1] Y. Arai: On axiom systems of propositional calculi. II. Proc. Japan Acad., **41**, 440-442 (1965).

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[3] Y. Imai and K. Iséki: On axiom systems of propositional calculi. I. Proc. Japan Acad., **41**, 436-439 (1965).

[4] K. Iséki: On axiom systems of propositional calculi. IV. Proc. Japan Acad., **41**, 575-577 (1965).