81. *On Axiom Systems of Propositional Calculi. XVIII*

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In their notes ([1], [2]), Y. Arai and K. Iséki discuss on some theses of equivalential calculus introduced by S. Leśniewski (see, [3]).

The fundamental axioms of equivalential calculus are given by

$E1$ *EEEprEqpErq*,

$E2$ *EEpEqrEEpqr*,

where E is the truth functor in the calculus (see, [4]). In his paper, Y. Arai has proved that the equivalential calculus characterizes the following theses:

(1) *EEpqEqp*, *EEEpqrEpEqr*,

and he deduced some theses in the equivalential calculus by using the inference rule of substitution and detachment: α and $E\alpha\beta$ imply β.

In this note, we shall show that *EEpqEEprErq*, the system (1) and the set of $E1$, $E2$ are equivalent. For the proof we shall use the prooflines by J. Lukasiewicz.

Proof. From the following fundamental thesis, i.e.,

1 *EEpqEEprErq*,

we have the following theses:

1 p/Epq, $q/EEpsEsq$ $*C1$ r/s—2,

2 *EEEpqrErEEpsEsq*.

2 p/Epq, q/r, $r/ErEEpsEsq$ $*C2$—3,

3 *EEEpqsEsEpq*.

3 $s/EEprErq$ $*C1$—4,

4 *EEEprErqEpq*.

2 p/Epr, q/Erq, r/Epq $*C4$—5,

5 *EEpqEEEprsEsErq*.

3 $s/EEEprsEsErq$ $*C5$—6,

6 *EEEEprsEsErqEpq*.

6 r/p, s/p, q/p $*C3q/p$, s/p—7,

7 *Epp*.

1 q/p $*C7$—8,

8 *EEprErp*.

5 q/p, r/q $*C7$—9,

9 *EEEpqsEsEqp*.

9 $s/EEprErq$ $*C1$—10,

10 *EEEprErqEqp*.

8 $p/EEprErq$, r/Eqp *C10—11,

11 $EEqpEEprErq$.

11 q/Eqp, $p/EEprErq$, r/s *C11—12,

12 $EEEEprErqsEsEqp$.

12 $s/EErqEpr$ *C8 p/Epr, r/Erq—13,

13 $EEErqEprEqp$.

8 $p/EErqEpr$, r/Eqp *C13—14,

14 $EEpqEErpEqr$.

14 p/Epq, $q/EErpEqr$, r/s *C14—15,

15 $EEsEpqEEErpEqrs$.

15 s/Epq, p/q, q/p *C8 r/q—16,

16 $EEErqEprEpq$.

8 $p/EErqEpr$, r/Epq *C16—17,

17 $EEpqEErqEpr$.

9 p/Epq, q/r, $s/ErEpq$ *C9—18,

18 $EErEqpErEpq$.

1 $p/EsEqp$, $q/EsEpq$, r/t *C18—19,

19 $EEEsEqptEtEsEpq$.

18 $r/EEsEqpt$, q/t, $p/EsEpq$ *C19—20,

20 $EEEsEqptEEsEpqt$.

20 s/Erq, q/p, p/r, t/Epq *C15—21,

21 $EEErqErpEpq$.

8 $p/EErqErp$, r/Epq *C21—22,

22 $EEpqEErqErp$.

18 r/Epq, q/Erq, p/Erp *C22—23,

23 $EEpqEErpErq$.

18 $r/EEpqs$, q/s, p/Eqp *C9—24,

24 $EEEpqsEEqps$.

4 $p/EErqr$, r/Erq *C4 p/Erq—25,

25 $EEErqrq$.

24 p/Erq, q/r, s/q *C25—26,

26 $EErErqq$.

20 s/r, q/r, p/q, t/q *C26—27,

27 $EErEqrq$.

24 p/r, q/Eqr, s/q *C27—28,

28 $EEEqrrq$.

8 $p/EEqrr$, r/q *C28—29,

29 $EqEEqrr$.

23 p/Epq, $q/EErpErq$, r/s *C23—30,

30 $EEsEpqEsEErpErq$.

30 s/q, p/Eqr, q/r, r/s *C29—31,

31 $EqEEsEqrEsr$.

31 $q/EqEEsEEqrEsr$, r/p, s/t $*C31$—32,

32 $EEtEEqEEsEEqrEsrpEtp$.

32 $t/EqErs$, q/r, r/s, s/q, $p/ErEqs$ $*C31$ $q/EqErs$, s/r, r/Eqs—33,

33 $EEqErsErEqs$.

18 p/Eqs, q/r, $r/EqErs$ $*C33$—34,

34 $EEqErsEEqrs$.

20 s/q, q/r, p/s, $t/EEqsr$ $*C34$—35,

35 $EEqEsrEEqsr$.

8 $p/EqEsr$, $r/EEqsr$ $*C35$—36,

36 $EEEqsrEqEsr$.

The theses 13 and 35 are the axioms by S. Leśniewski. The theses 8 and 36 are (1).

Therefore, from the results proved in [1], [2] and in this note, we have the following theorem.

Theorem. *The equivalential calculus is characterized by the axiom* 1: $EEpqEEprErq$.

Therefore, the following theses are equivalent:

1) $EEEprEEqpErq$, $EEpEqrEEpqr$,
2) $EEpqEqp$, $EEEpqrEpEqr$,
3) $EEpqEErqEpr$,
4) $EEpqEEprErq$.

References

[1] Y. Arai: On axiom systems of propositional calculi. XVII. Proc. Japan Acad., **42**, 351-354 (1966).

[2] K. Iséki: On axiom systems of propositional calculi. XV. Proc. Japan Acad., **42**, 217-220 (1966).

[3] S. Leśniewski: Grundzüge eines neuen Systems der Grundlagen der Mathematik. Fund. Math., **14**, 1-81 (1929).

[4] A. N. Prior: Formal Logic. Oxford (1962).