

44. On Axiom Systems of Propositional Calculi. XXV

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In this paper, we shall show that the following new system:

- 1 $CCpqCCqrCpr,$
- 2 $CCCpqqp,$
- 3 $CpCCpqq,$

is equivalent to Tarski-Bernays axiom system.

In their papers, [1], [2], Y. Imai and K. Iséki have proved that Tarski-Bernays axiom system implies $CpCCpqq$ and other systems by using their algebraic formulation. Therefore we shall prove that the above system implies Tarski-Bernays axiom system.

- 1 $p/Cpq, q/CCqrCpr, r/s *C1-4,$
- 4 $CCCCqrCprCpqs.$
4 $q/Cqr, r/Csr, s/CCsqCpCsr *C4 p/s, s/CpCsr-5,$
- 5 $CCpCqrCCsqCpCsr.$
4 $s/CCCprCqrs *C1 p/Cqr, q/Cpr, r/s-6,$
- 6 $CCpCqCCprCqrs.$
1 $q/CCpqq *C3-7,$
- 7 $CCCCpqqCpr.$
7 $r/CCCCpqqCCpp *C6 p/Cpq, r/p, s/p-8,$
- 8 $CpCCCCpqqCCpp.$
5 $p/q, q/Cqr, s/p *C3 p/q, q/r-9,$
- 9 $CCpCqrCqCpr.$
9 $q/CCCpqqp, r/CCppp *C8-C2-10,$
- 10 $CpCCppp.$
5 $q/Cqp, r/p *C10-11,$
- 11 $CCsCqpCpCsp.$
11 $s/q, q/Cqp *C3 p/q, q/p-12,$
- 12 $CpCqp.$

Theses 1, 2, and 12 are axioms of Tarski-Bernays system. Hence the proof is complete.

It is easily seen that $CCpqCCqrCpr$, $CCCpqqp$, $CpCCpqq$, and COp are equivalent to classical propositional calculus. O is propositional constant, and we define CpO as Np . This follows from C. A. Meredith [3].

References

- [1] K. Iséki: Algebraic formulation of propositional calculi. Proc. Japan Acad., **41**, 803-807 (1965).
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- [3] C. A. Meredith: Single axioms for the systems (C, N) , (C, O) , and (A, N) of the two-valued propositional calculus. Journal of Computing Systems, **3**, 155-164 (1953).