202. *Note on Lie Subrings in Malcev Rings*

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A Malcev ring or a Moufang-Lie ring [2] is an anti-commutative ring M satisfying

$$(xy)(zx)+(xy\cdot z)x+(yz\cdot x)x+(zx\cdot x)y=0 \text{ for all } x, y, z\in M.$$

Any Lie ring is Malcev and there are non-Lie Malcev rings [3, § 3]. A Jacobian J of x, y, z is a skew-symmetric function defined by

$$J(x, y, z)=(xy)z+(yz)x+(zx)y.$$

In [3], Sagle proved the following theorem:
Let A, B, C be subsets of a Malcev algebra M. If it holds

$$J(A, A, M)=J(B, B, M)=J(C, C, M)=J(A, B, C)=(0),$$

then there is a Lie subalgebra of M containing the subset $A\cup B\cup C$.

In this note, we remark that the method by Grätzer and Schmidt [1] for proof of associativity theorem for alternative rings can be also applied for Malcev rings and obtain the following similar result, which is a generalization of Sagle's theorem:

Theorem. *Let $A_1, A_2, \cdots, A_n (n\geqq 2)$ be subsets of a Malcev ring M and D^* be Malcev subring of M generated by $D=\bigcup_{i=1}^n A_i$. Then D^* is a Lie subring of M if and only if*

$$J(A_i, A_i, D^*)=(0) \tag{1}$$

and

$$J(A_i, A_j, A_{ij})=(0), \qquad i, j=1, \cdots, n, \tag{2}$$

where A_{ij} means a set of products of at most $n-2$ factors a_{k_m} such that $a_{k_m}\in A_{k_m}$, k_m's are different each other and $k_m\neq i$, $k_m\neq j$ for $n>2$ and $i\neq j$, and $A_{ij}=(0)$ for $n=2$ or $i=j$.

Proof. We assume (1) and (2) and prove that $J(D^*, D^*, D^*)=(0)$. First $J(D, D, D)\subseteqq \bigcup_{i,j,k} J(A_i, A_j, A_k)=(0)$ by (1) and (2). Denote D^p a set of products of p elements of D and by induction assume $J(D^p, D^q, D^r)=(0)$ has been proved for all p, q, r satisfying $p+q+r<N$, p, q, r being positive integers. Now by [3, (2.7)] we have

$$J(aa', b, c)+J(a, b, a'c)=J(a', b, c)a+J(a, a', b)c.$$

Hence $J(a', b, c)=J(a, a', b)=0$ implies $J(aa', b, c)=J(a, b, ca')$. Let $p+q+r=N$, by the assumption of induction, any element $J(a, b, c)$ of $J(D^p, D^q, D^r)$ can be changed to the form

$$J(a_i, b, c'), \qquad c' \in D^{r+p-1},$$

and by repeating this process for b, to the form

$$J(a_i, a_j, c''), \qquad c'' \in D^{N-2}.$$

Hence if $i=j$ this term vanishes by (1), and we may assume that $i \neq j$ and $c'' \in A_{ij}$, then such term also vanishes from (2), and $J(D^p, D^q, D^r)=(0)$ for $p+q+r=N$. The converse is clear.

References

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