13. *On Axioms of Ontology*

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It is well known that the following expression can act the only axiom of ontology [1], [2]:

(α) $x \varepsilon X \equiv [\exists y]\{y \varepsilon x \wedge y \varepsilon X\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$.

The proof of this based on the following axiom of ontology has been given in "S. Leśniewski's Calculus of Names" by J. Slupecki [2]:

$T1.1.$ $x \varepsilon X \equiv [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\} \wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$.

In this paper we shall give *the proof of* $T1.1$ *based on* (α).

The proofs of theorems will be given in the form of suppositional proofs used by J. Slupecki.

(I) $x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon x$.

Proof.

(1)	$x \varepsilon X$	{premise}
(2)	$y \varepsilon x$	
(3)	$y \varepsilon x \wedge y \varepsilon x$	{2}
(4)	$[\exists y]\{y \varepsilon x \wedge y \varepsilon x\}$	{$D\Sigma$: 3}
(5)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{α, 1}
	$x \varepsilon x$	{α, 4, 5}

(II) $x \varepsilon X \wedge y \varepsilon x \supset x \varepsilon y$.

Proof.

(1)	$x \varepsilon X$	{premise}
(2)	$y \varepsilon x$	
(3)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{α, 1}
(4)	$x \varepsilon x \wedge y \varepsilon x \supset x \varepsilon y$	{$O\Pi$: 3}
(5)	$x \varepsilon x$	{I, 1, 2}
	$x \varepsilon y$	{4, 5, 2}

(III) $x \varepsilon X \wedge y \varepsilon x \supset y \varepsilon X$.

Proof.

(1)	$x \varepsilon X$	{premise}
(2)	$y \varepsilon x$	
(3)	$[x, z]\{x \varepsilon y \wedge z \varepsilon y \supset x \varepsilon z\}$	{α, 2}
(4)	$x \varepsilon y$	{II, 1, 2}
(5)	$x \varepsilon y \wedge x \varepsilon X$	{4, 1}
(6)	$[\exists x]\{x \varepsilon y \wedge x \varepsilon X\}$	{$D\Sigma$: 5}
	$y \varepsilon x$	{α, 6, 3}

(IV) $x \varepsilon X \supset [y]\{y \varepsilon x \supset y \varepsilon X\}$.

Proof.

(1)	$x \varepsilon X$	{premise}
(2)	$y \varepsilon x \supset y \varepsilon X$	{III, 1}
	$[y]\{y \varepsilon x \supset y \varepsilon X\}$	{$D\Pi$: 2}

(V) $x \varepsilon X \supset [\exists y]\{y \varepsilon x\}$.

Proof.

(1)	$x \varepsilon X$	{premise}

(2)	$[\exists y]\{y \varepsilon x \wedge y \varepsilon X\}$	$\{\alpha, 1\}$
(3)	$y_1 \varepsilon x \wedge y_1 \varepsilon X$	$\{O\Sigma: 2\}$
(4)	$y_1 \varepsilon x$	$\{3\}$
	$[\exists y]\{y \varepsilon x\}$	$\{D\Sigma: 4\}$

(VI) $x \varepsilon X \supset [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$
$\wedge [y]\{y \varepsilon x \supset y \varepsilon X)$. $\{V, \alpha, \text{IV}\}$

(VII) $[\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$
$\wedge [y]\{y \varepsilon x \supset y \varepsilon X\} \supset x \varepsilon X$.

Proof.

(1)	$[\exists y]\{y \varepsilon x\}$	{premise}
(2)	$[y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$	{premise}
(3)	$[y]\{y \varepsilon x \supset y \varepsilon X\}$	{premise}
(4)	$y_1 \varepsilon x$	$\{O\Sigma: 1\}$
(5)	$y_1 \varepsilon x \wedge y_1 \varepsilon x$	$\{4\}$
(6)	$[\exists y]\{y \varepsilon x \wedge y \varepsilon x\}$	$\{O\Sigma: 5\}$
(7)	$x \varepsilon x$	$\{\alpha, 6, 2\}$
(8)	$x \varepsilon x \wedge y_1 \varepsilon x \supset x \varepsilon y_1$	$\{O\Pi: 2\}$
(9)	$x \varepsilon y_1$	$\{8, 7, 4\}$
(10)	$y_1 \varepsilon x \supset y_1 \varepsilon X$	$\{O\Pi: 3\}$
(11)	$y_1 \varepsilon X$	$\{10, 4\}$
	$x \varepsilon X$	$\{\text{III}, 11, 9\}$

(VIII) $x \varepsilon X \equiv [\exists y]\{y \varepsilon x\} \wedge [y, z]\{y \varepsilon x \wedge z \varepsilon x \supset y \varepsilon z\}$
$\wedge [y]\{y \varepsilon x \supset y \varepsilon X\}$. $\{\text{VI, VII}\}$

The proof is complete and it does not involve the rule of extensionality.

References

[1] C. Lejewski: On Lesniewski's ontology. Ratio, **1**, 150–176 (1958).
[2] J. Slupecki: S. Lesniewski's calculus of names. Studia Logica (Warszawa), **3** (1955).