110. *On the Alexander-Pontrjagin Duality Theorem*

By Hikosaburo KOMATSU
Department of Mathematics, University of Tokyo

(Comm. by Kunihiko KODAIRA, M. J. A., June 12, 1968)

Let V be an open set in $\boldsymbol{R}^n$ and let K be a compact subset of V. In [2] we proved the duality between $H^{n-p}(K, \boldsymbol{C})$ and $H^p_K(V, \boldsymbol{C}) = H^p(V \bmod V-K, \boldsymbol{C})$, $p=0, 1, \cdots, n$, under the assumption that $\dim H^{n-p}(K, \boldsymbol{C})$ is at most countable for $p=0, 1, \cdots, n$. The purpose of this note is to show that the assumption holds unconditionally and therefore that the duality holds for any compact set K.

Theorem 1. *Let K be a compact set in $\boldsymbol{R}^n$ and let F be a field. Then the dimension of the cohomology group $H^p(K, F)$ (defined as in Godement* [1]) *is at most countable for any p.*

Proof. Since

$$H^p(K, F) = \varinjlim H^p(U, F)$$

when U runs over all neighborhoods of K by Théorème 4.11.1 of [1], it suffices to show that there exists a countable fundamental system of neighborhoods of K consisting of open sets U_j such that $\dim H^p(U_j, F) < \infty$.

Clearly we can find a countable fundamental system of neighborhoods of K. Let V be a member. At each point $x \in K$, there is an open ball W_x containing x and contained in V. Choose a finite subcovering W_i of the covering $\{W_x ; x \in K\}$ and let $U = \cup W_i$. If we denote by $\mathcal{W}$ the open covering $\{W_i\}$ of U, it follows from Leray's theorem (Théorème 5.2.4 of [1]) that $H^p(U, F)$ is isomorphic to the cohomology group $H^p(\mathcal{W}, F)$ of the covering $\mathcal{W}$. The latter is clearly of finite dimension. Thus there is an open set U which satisfies $K \subset U \subset V$ and $\dim H^p(U, F) < \infty$.

Now, combining Theorem 1 with Theorem 11 of [2] (cf. also Theorem 20 (ii) of [3]), we obtain the following Alexander-Pontrjagin duality theorem.

Theorem 2. *Let K and V be as in Theorem* 1. *Then $H^{n-p}(K, \boldsymbol{C})$ and $H^p_K(V, \boldsymbol{C})$ have the natural structure of the dual Fréchet-Schwartz space and of the Fréchet-Schwartz space, respectively, and they are the strong dual spaces of each other. More precisely there is an at most countable cardinal number b^{n-p} such that $H^{n-p}(K, \boldsymbol{C}) \cong \boldsymbol{C}^{(b^{n-p})}$ and $H^p_K(V, \boldsymbol{C}) \cong \boldsymbol{C}^{b^{n-p}}$.*

Consequently, the Jordan-Brouwer theorem (Theorem 12) of [2] is improved as follows:

Theorem 3. *Let V, K, and b^{n-1} be as in Theorem 2. Then, the number of connected components of $V-K$ is equal to the sum of b^{n-1} and the number of connected components of V.*

References

[1] R. Godement: Topologie algébrique et théorie des faisceaux. Hermann (1958).
[2] H. Komatsu: Resolutions by hyperfunctions of sheaves of solutions of differential equations with constant coefficients. Math. Ann., **176**, 77-86 (1968).
[3] ——: Projective and injective limits of weakly compact sequences of locally convex spaces. J. Math. Soc. Japan, **19**, 366-383 (1967).