

## 116. On Fatou- and Plessner-Type Theorems

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This is a résumé of parts of my papers [4]–[6] published or unpublished. We give explanations to the diagramme.

$f$  is a “map” from  $U$  into  $\Omega$ .

Fatou points, Meier points and Plessner points for  $f$  are defined suitably on the boundary  $\partial U$  of  $U$ .

$F$  is Fatou-type theorem: If  $f$  is bounded in  $U$ , then almost every point of  $\partial U$  is a Fatou point. Here, “almost every” means “except for a set of Lebesgue measure zero on  $\partial U$ ”.

$P$  is Plessner-type theorem: Almost every point of  $\partial U$  is either a Plessner point or a Fatou point.

$MF$  is Topological Fatou-type theorem: If  $f$  is bounded in  $U$ , then nearly every point of  $\partial U$  is a Meier point. Here, “nearly every” means “except for a set of first Baire category on  $\partial U$ ”.

$MP$  is Topological Plessner-type theorem: Nearly every point of  $\partial U$  is either a Plessner point or a Meier point.

Diagramme

$U$	Open disk $ z  < 1$ in $R^2$			Open unit ball in $R^3$	
$\Omega$	Riemann sphere $ w  \leq \infty$			$R^1$ added $-\infty$ and $+\infty$	
$f$	Meromorphic function	Quasi-conformal function	Algebroid function	Real-harmonic function	
$F$	Yes <sup>1)</sup>	No <sup>5)</sup>	Yes <sup>8)</sup>	Yes <sup>1)</sup>	Yes <sup>1)</sup>
$P$	Yes <sup>2)</sup>	No <sup>5)</sup>	Yes <sup>8)</sup>	Yes <sup>11)</sup>	Yes <sup>12)</sup>
$MF$	Yes <sup>3)</sup>	Yes <sup>6)</sup>	Yes <sup>9)</sup>	Yes <sup>11)</sup>	Yes <sup>13)</sup>
$MP$	Yes <sup>4)</sup>	Yes <sup>7)</sup>	Yes <sup>10)</sup>	Yes <sup>11)</sup>	Yes <sup>14)</sup>

Notes. 1) This is now classical. 2) Plessner [3], Satz I. 3) Meier [1], Satz 6. 4) Meier [1], Satz 5. 5) Noshiro [2], p. 119. 6) Yamashita [4], Theorem 1. 7) Yamashita [4], Theorem 2. 8) Yamashita [5], Theorem 1. 9) Yamashita [5], Theorem 3. 10) Yamashita [5], Theorem 2. 11) This is proved implicitly in [6]. 12) Yamashita [6], Theorem 2. 13) Yamashita [6], Theorem 3. 14) Yamashita [6], Theorem 1.

## References

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