44. Notes on Regular Semigroups. III

By Sándor LAJOS
K. Marx University of Economics, Budapest, Hungary

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Following the notation and terminology of A. H. Clifford and G. B. Preston [1] we announce some further results concerning regular semigroups.

Theorem 1. *For a semigroup S the following conditions are mutually equivalent:*

(A) *S is regular.*

(B) *$L \cap R = RSL$ for every left ideal L and every right ideal R of S.*

(C) *$R(a) \cap L(b) = R(a)SL(b)$ for every couple of elements in S.*

(D) *$L(a) \cap R(a) = R(a)SL(a)$ for each element a of S.*

(E) *$B(a) = R(a)SL(a)$ for every element a of S.*

Notation. $L(a)$, $R(a)$, and $B(a)$ denote the principal left, right, and bi-ideal of S generated by the element a of S, respectively.

An element a of a semigroup S is called (m, n)-regular if there exists x in S such that $a^m x a^n = a$. A semigroup S is said to be duo if every one-sided ideal of S is two-sided.

Theorem 2. *For a semigroup S the following statements are pairwise equivalent:*

(i) *S is a completely regular duo semigroup.*

(ii) *S is $(2, 2)$-regular and duo.*

(iii) *S is $(2, 1)$-regular and duo.*

(iv) *S is $(1, 2)$-regular and duo.*

(v) *S is a regular duo semigroup.*

(vi) *S is a completely regular inverse semigroup.*

(vii) *S is a semilattice of groups.*

(viii) *S is regular and $LR = RL$ for every left ideal L and every right ideal R of S.*

(ix) *S is centric and every principal ideal is globally idempotent.*

(x) *S is duo and each principal ideal of S is globally idempotent.*

The following result gives various ideal-theoretic characterizations of semigroups which are semilattices of groups.

Theorem 3. *For a semigroup S the following conditions are equivalent:*

(1) *S is a semilattice of groups.*

(2) *$B \cap B' = BB'$ for every couple of bi-ideals in S.*

(3) *$B \cap B' = BSB'$ for every couple of bi-ideals in S.*

(4) $B \cap B' = SBB'S$ *for every couple of bi-ideals in* S.

(5) $B \cap Q = SBQS$ *for every bi-ideal* B *and every quasi-ideal* Q *of* S.

(6) $B \cap Q = SQBS$ *for each bi-ideal* B *and each quasi-ideal* Q *of* S.

(7) $Q \cap Q' = SQQ'S$ *for every couple of quasi-ideals in* S.

(8) $B \cap L = SLBS$ *for every bi-ideal* B *and every left ideal* L *of* S.

(9) $B \cap R = SBRS$ *for each bi-ideal* B *and each right ideal* R*of* S.

(10) $L \cap Q = SLQS$ *for every left ideal* L *and every quasi-ideal* Q *of* S.

(11) $Q \cap R = SQRS$ *for any quasi-ideal* Q *and any right ideal* R *of* S.

(12) $L \cap R = SLRS$ *for every left ideal* L *and every right ideal* R *of* S.

References

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