

## 88. A New Characterization of Regular Duo Semigroups

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Let  $S$  be a semigroup. Following the notation and terminology of A. H. Clifford and G. B. Preston [1] we say that  $S$  is regular if, to every element  $a$  in  $S$ , there exists at least one element  $x$  in  $S$  such that  $axa = a$ . For the sake of brevity we shall say that  $S$  is a duo semigroup if every one-sided ideal of  $S$  is two-sided. The author proved several ideal-theoretic characterizations of regular duo semigroups (cf. [2]-[4]).

In this short note some new criteria for a semigroup to be a regular duo semigroup will be proved.

**Theorem 1.** *A semigroup  $S$  is a regular duo semigroup if and only if the conditions*

$$(1) \quad (L \cup LS)^2 = L,$$

and

$$(2) \quad (R \cup SR)^2 = R$$

hold for every left and every right ideal of  $S$ , respectively.

**Proof.** Let  $S$  be a regular duo semigroup. Then every one-sided ideal of  $S$  is two-sided, and

$$(3) \quad I \cap J = IJ$$

holds for any couple of ideals of  $S$ . (3) implies that every ideal  $I$  of  $S$  is globally idempotent, i.e.

$$(4) \quad I^2 = I$$

for any ideal  $I$  of  $S$ . This implies both (1) and (2).

Conversely, suppose that  $S$  is a semigroup with properties (1) and (2) for every left and right ideal, respectively. Then (2) implies that each right ideal  $R$  of  $S$  is also a left ideal, and (1) implies that every left ideal  $L$  of  $S$  is two-sided. Therefore  $S$  is a duo semigroup. Finally (1) implies (4) for any ideal  $I$  of  $S$ , which is equivalent to the regularity of a duo semigroup. (See [5].)

Theorem 1 is completely proved.

Next utilizing the author's recent results concerning the  $(m, n)$ -ideals of regular duo semigroups (see [6]), one can prove the following result.

**Theorem 2.** *A semigroup  $S$  is a regular duo semigroup if and only if the relation*

$$(5) \quad (B \cup SB)^2 = B = (B \cup BS)^2$$

holds for any bi-ideal  $B$  of  $S$ .

It is easy to see that the result of Theorem 2 remains true with quasi-ideal instead of bi-ideal.

**Theorem 3.** *A semigroup  $S$  is a regular duo semigroup if and only if the condition*

$$(6) \quad (Q \cup QS)^2 = Q = (Q \cup SQ)^2$$

*holds for every quasi-ideal  $Q$  of  $S$ .*

### References

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