31. *The Method of Successive Approximation in the Old Japanese Mathematics.*

By Matsusaburo FUJIWARA, M.I.A.
Mathematical Institute, Tohoku Imperial University, Sendai.

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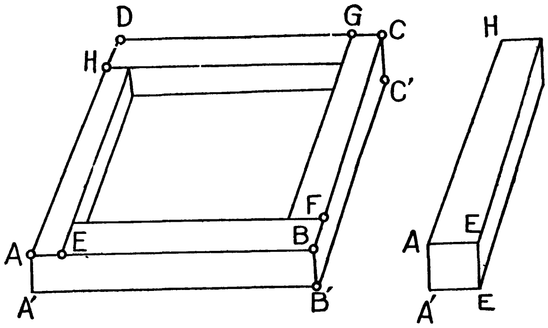
It is well known that we can find the method of successive approximations in a manuscript Kaihô-Yeizikuzitu (開方盈朒術) by *Genzyun Nakane* (中根彥循, 1701–1761), written in 1729 (享保十四年). It was remarked however by Mr. Yoshio Mikami[1] that the said method was already used by *Takakazu Seki* (關孝和) in his manuscript Daizitu-Bengi (題術辨議), the date unknown.

I wish here to report that a work Sangaku-Yenteiki (算學淵底記) or Sanpô Hutudankai (算法勿憚改) by *Murase* (村瀬義益), written in 1673 (寛文十三年), contains two problems solved by the method of successive approximations.

The first problem treats of a fireplace-frame (爐縁), which consists of 4 pieces of rectangular parallelopiped, whose breadth and height are equal. The said problem runs as follows.



Given the volume of a fireplace-frame $v=192$ and the length $AB=14$, it is required to find the length $AA'=BB'=AE=BF=\cdots\cdots$

If we denote $AA'=x$, then we have a cubic equation

$$x^2(14-x)=\frac{1}{4}\times 192=48\,.$$

The author of the said work gave two methods. The first starts from the form $x=\sqrt{\frac{1}{14}(48+x^3)}$ and the second from $x=\sqrt{\frac{48}{14-x}}$. Then determining $x_1, x_2, x_3, \cdots\cdots$ successively by

$$x_1=\sqrt{\frac{1}{14}(48+x_0^3)}\,,\quad x_2=\sqrt{\frac{1}{14}(48+x_1^3)}\,,\quad x_3=\sqrt{\frac{1}{14}(48+x_2^3)}\,,\cdots\cdots;$$

$$x_1=\sqrt{\frac{48}{14-x_0}}\,,\qquad x_2=\sqrt{\frac{48}{14-x_1}}\,,\qquad x_3=\sqrt{\frac{48}{14-x_2}}\,,\cdots\cdots$$

respectively (starting with $x_0=0$), we obtain

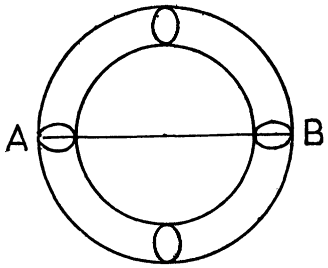
$$x_1=1.85\,,\quad x_2=1.97\,,\quad x_3=1.9936\,;$$

$$x_1=1.85\,,\quad x_2=1.9876\,,\quad x_3=1.99907$$

1) Tôyô-Gakuhô, vol. 21, 1934.

respectively. The author concluded from these results $x=2$ is the required answer, and also verified it by substituting $x=2$ in the given equation.

The second problem runs as follows.



Given an anchor-ring, whose volume is equal to 118.43540672 and the breadth AB is 14, it is required to find the radius of the circular section.

This problem can be reduced to the same cubic equation

$$x^2(14-x)=48,$$

when we use $\pi=3.1416$, as the author did.

It is known that *Takakazu Seki* has written a work which contains the solutions of problems proposed in Sangaku-Yenteiki ; so it is certain that this work is earlier than Seki's work considered above.