PAPERS COMMUNICATED

104. *The Electron Velocity Distribution in the Celestial Gaseous Assemblies in Radiative Equilibrium.*

By Yusuke HAGIHARA, M.I.A.
Astronomical Department, Tokyo Imperial University.

(Comm. Oct. 12, 1944.)

§ 1. It is known[1] that the necessary and sufficient condition for detailed balancing in a gaseous assembly consisting of atoms and light quanta is that the velocity distribution of the free particles is Maxwellian and that the distributions of the atoms in various quantum states and of the light quanta in various frequencies are respectively Boltzmann's and Planck's. In the physical state of the gaseous clouds and nebulae, such as of the planetary nebulae or of the diffuse matter in the interstellar space, the distribution of the light quanta is not Planck's and moreover, as has been shown elsewhere[2], that of the atoms in various quantum states is not Boltzmann's, so that it is not in the state of detailed balancing and hence it is not in thermodynamical equilibrium. Hence the physical condition in such gaseous assemblies ought to be sought for otherwise than in the state under laboratory conditions supposed to be in thermodynamical equilibrium. It has been shown in the previous works[3] of the present author that the extremely rarefied gaseous assemblies consisting of hydrogen atoms, hydrogen ions and free electrons, and even the assemblies containing oxygen, nitrogen and carbon in addition to hydrogen, and exposed to highly diluted high frequency radiation field, such as in the planetary nebulae, can be in a steady state in which the velocity distribution of the free electrons is not Maxwellian, although there is no transport phenomenon of particles as is usually treated in the kinetic theory of gases. The circumstance has been shown to be the same in the planetary nebulae of moderate optical thickness by solving the complicated problem of radiative transfer through the nebular layers, and the deviation of the electron velocity distribution from the Maxwellian has been seen to be moderately great according to the circumstances. It is interesting to compare this result with the theories[4] of Schrödinger,

---

[1] Dirac, Proc. Roy. Soc., A **106** (1924), 581; Fowler, Statistical Mechanics, 1916, 667.

[2] Hagihara, Jap. J. Astr. Geophys., **15** (1938), 1; Hagihara and Hatanaka, *ibid.*, **19** (1942), 135.

[3] Hagihara, Jap. J. Astr. Geophys., **17** (1940), 199; **17** (1940), 417; **18** (1940), 89; Hagihara and Soma, *ibid.*, **18** (1941), 149; Hagihara, *ibid.*, **19** (1941), 9, 75; **20** (1943), 1. 37; Hagihara and Soma, *ibid.*, **20** (1943), 59; Soma, *ibid.*, **20** (1943), 67; also Hagihara, Monthly Notices Roy. Astr. Soc., **100** (1940), 631.

[4] Schrödinger, Sitzungsber. Akad. Wiss. Wien. Math.-Phys. Kl., **121** (1912), 1305; Van Leeuwen, Journ. de Phys., **2** (1921), 374; Van Vleck, Electric and Magnetic Susceptibilities. 1932, 97; Cowling, Monthly Notices Roy. Astr. Soc., **92** (1932), 403.

van Leeuwen and Cowling on the Maxwellian velocity distribution of free electrons moving in a magnetic field.

In the present note I propose to report a brief summary and retrospection of the main results I have been able to reach on the electron velocity distribution in the celestial gaseous assemblies in radiative equilibrium by general discussions based on Hilbert's theorem[5] on summational invariants in the kinetic theory of gases, and also to indicate the direction for a further procedure we should take as soon as more accurate spectrophotometric measurements of such celestial gaseous assemblies and more extensive quantum-mechanical computations for the transition probabilities are available in future.

§ 2. Consider an extremely rarefied isotropic and homogeneous gaseous assembly in steady state consisting of free electrons, hydrogen atoms, hydrogen ions and several other types of atoms and ions, and exposed to diluted high frequency radiation incident to it. Hydrogen is supposed to be by far the most abundant among the rest of the elements present in the assembly, as is usual in most of the celestial objects. Denote by N_e and N_{lmn} respectively the numbers per unit volume of free electrons and of the m-ply ionised ions of the n-th excited state of the l-th element, the o-th element being hydrogen and the o-th excited state the ground state, the o-ply ionised ion the neutral atom. Then the Boltzmann equation for the frequency function f of the electron velocity distribution is written in the form:

$$\frac{D(N_e f)}{Dt}=\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{enc},1}+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{enc},2}+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{ff}}+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{abs}}$$
$$-\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{emis}}+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{coll}}, \tag{1}$$

with
$$\frac{DN_e}{Dt}=\iiint\left\{\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{ff}}+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{abs}}-\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{emis}}\right.$$
$$\left.+\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{coll}}\right\}du\,dv\,dw, \tag{2}$$

where the terms on the right hand side of (1) represent respectively in the order of succession the rates of increase in the number of free electrons per unit volume in the velocity range u, v and w to $u+du$, $v+dv$ and $w+dw$, due to the encounters between two free electrons, due to the encounters between a free electron and an ion, or an atom, due to the free-free radiative transitions, due to the ionisations by absorption of radiation, due to the captures of free electrons to ions with emission of radiation, and due to the inelastic and superelastic collisions with or without electron exchange.

---

[5] Hilbert, Math. Ann. **72** (1912), 562; Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen, 1913, Kap. XXII; Jeans, The Dynamical Theory of Gases, 1921, Chap. VIII; Chapman and Cowling, The Mathematical Theory of Non-Uniform Gases, 1939, p. 50 and Chap. VII; Enskog, Kinetische Theorie der Vorgänge in mässig verdünnten Gasen, 1917, s. 14 u. s. 27.

We expand the various terms on the right hand side of (1), together with f, in series of the Hermite polynomials in the form:

$$f(u,v,w)=\frac{1}{\alpha^3}\sum_{i,j,k}\beta_{ijk}\varphi_{ijk}\,, \tag{3}$$

$$\left.\begin{aligned}
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{enc},1}&=\frac{N_e^2}{\alpha^3}\sum_{i,j,k}V_{ijk}\varphi_{ijk}\,,\\
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{enc},2}&=\sum_{l,m,n}\frac{N_eN_{lmn}}{\alpha^3}\sum_{i,j,k}V_{ijk}^{(lmn)}\varphi_{ijk}\,,\\
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{ff}}&=\sum_{l,m,n}\frac{N_eN_{lmn}}{\alpha^3}\sum_{i,j,k}F_{ijk}^{(lmn)}\varphi_{ijk}\,,\\
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{emis}}&=\sum_{l,m,n}\frac{N_eN_{lmn}}{\alpha^3}\sum_{i,j,k}E_{ijk}^{(lmn)}\varphi_{ijk}\,,\\
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{abs}}&=\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}\sum_{i,j,k}A_{ijk}^{(lmn)}\varphi_{ijk}\,,\\
\left[\frac{\partial}{\partial t}(N_e f)\right]_{\mathrm{coll}}&=\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}\sum_{i,j,k}K_{ijk}^{(lmn)}\varphi_{ijk}\,,
\end{aligned}\right\} \tag{4}$$

where $\varphi_{ijk}=(-1)^{i+j+k}H_{ijk}\varphi_{000}\,,$

$$H_{ijk}=H_i\!\left(\frac{u}{\alpha}\right)H_j\!\left(\frac{v}{\alpha}\right)H_k\!\left(\frac{w}{\alpha}\right),\qquad \varphi_{000}=(2\pi)^{-\frac{3}{2}}e^{-(u^2+v^2+w^2)/2\alpha^2}\,,$$

$$H_i(U)=U^i-\frac{i(i-1)}{2\cdot 1!}U^{i-2}+\frac{i(i-1)(i-2)(i-3)}{2^2\cdot 3!}U^{i-4}+\cdots$$

When we substitute (3) and (4) in (1) and equate the coefficients of the same φ_{ijk} for various combinations of positive integral values of i,j and k, including zero, we get

$$\begin{aligned}
\frac{D}{Dt}\left(\frac{\beta_{ijk}}{\alpha^3}\right)=&\frac{N_e}{\alpha^3}V_{ijk}+\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}V_{ijk}^{(lmn)}+\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}F_{ijk}^{(lmn)}\\
&+\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}A_{ijk}^{(lmn)}-\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3}E_{ijk}^{(lmn)}+\sum_{l,m,n}\frac{N_{lmn}}{\alpha^3N_e}K_{ijk}^{(lmn)}\\
&-\frac{\beta_{ijk}}{\alpha^3}\frac{1}{N_e}\frac{DN_e}{Dt}\,,
\end{aligned} \tag{5}$$

$$(i,j,k=0,1,2,\ldots)\,.$$

For a steady state we have $Df/Dt=0$, or

$$\frac{D}{Dt}\left(\frac{\beta_{ijk}}{\alpha^3}\right)=0\,,\qquad (i,j,k=0,1,2,\ldots)\,. \tag{6}$$

For a steady state of the total number of free electrons we should have

$$\frac{DN_e}{Dt}=0\,. \tag{7}$$

$A_{ijk}^{(lmn)}$'s contain the spectral intensity distribution $I(\nu)$ of the incident radiation and are independent of the frequency function f, while the coefficients in the expansions are functions of β_{ijk}'s but independent of $I(\nu)$. Hence the values of β_{ijk}'s are generally determined by (5) successively and there are in general non-vanishing values of β_{ijk}'s besides $\beta_{000}=1$, satisfying the condition (6). Thus there can exist steady states in which the electron velocity distribution is not Maxwellian.

Boltzmann's equations for the frequency functions of the velocity distribution for the ions can be written down analogously.

If we impose further the stationariness condition that the number N_{lmn} is constant for every combination of l, m, n, then N_{lmn}'s are completely determined by the cyclic equations expressing the stationariness condition for each state of each ion of each element. However $N_0=\sum_{m,n} N_{0mn}$ and the abundance ratios $\gamma_l=(\sum_{m,n} N_{lmn})/(\sum_{m,n} N_{0mn})$ are left undetermined, which should be determined on the basis of the spectrophotometric observations of the celestial object.

Now it is known from the kinetic theory of gases by the name of Hilbert's theorem that there exist five functions of fundamental importance called the summational invariants which remain unaltered in any simple encounter. For a binary encounter of an ion of the type lmn with a free electron they are

$$1,\ mu+m_{lmn}u_{lmn}\,,\quad mv+m_{lmn}v_{lmn}\,,\quad mw+m_{lmn}w_{lmn}\,,$$

and

$$m(u^2+v^2+w^2)+m_{lmn}(u_{lmn}^2+v_{lmn}^2+w_{lmn}^2)\,,$$

where m_{lmn} is the mass of the ion and u_{lmn}, v_{lmn} and w_{lmn} are its velocity components.

We assume that the gaseous assembly is isotropic and homogeneous and has no mass motion as a whole and finally that the total kinetic energy of the free electrons is constant in time. The last condition has been proved to be the same as the condition for radiative equilibrium, when the total number of atoms other than hydrogen is very small compared with the number of free electrons, as is usual in any celestial gaseous assembly. The first and the second conditions, together with Hilbert's theorem, state that $ma^2\equiv kT_\varepsilon=$constant and that $\beta_{100}=\beta_{010}=\beta_{001}=0$. The third condition, that is nothing but the condition for radiative equilibrium, is written, by being combined with Hilbert's theorem and our Boltzmann equations, in the form:

$$\iiint\left\{\left[\frac{\partial}{\partial t}(N_ef)\right]_{\rm abs}-\left[\frac{\partial}{\partial t}(N_ef)\right]_{\rm emis}+\left[\frac{\partial}{\partial t}(N_ef)\right]_{\rm ff}\right.$$
$$\left.+\left[\frac{\partial}{\partial t}(N_ef)\right]_{\rm coll}\right\}(H_{200}+H_{020}+H_{002})dudvdw=0\,. \tag{8}$$

The functions in the integrand contain $\beta_{200}=\beta_{020}=\beta_{002}\equiv\beta_2$ and $I(\nu)$, and also the abundance ratios γ_l's. Hence by this equation we can determine β_2 when $I(\nu)$, T_ε and γ_l's are previously given.

§ 3. Suppose that our gaseous assembly consists of free electrons, hydrogen atoms, hydrogen ions and OIII ions. The transition probabilities for hydrogen are taken from Gaunt's wave-mechanical computation[6].

$$\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{abs}}=\frac{C_1 N_{010} m}{4\pi h V}\left[\frac{I(\nu)}{\nu^4}\right]_{\nu=\nu_1+\frac{m}{2h}V^2}, \qquad V^2=u^2+v^2+w^2,$$

$$\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{emis}}=N_e N_{010} C_2\cdot\sum_{s=1}^{\infty}\frac{f}{s^3 V\left(\nu_s+\frac{m}{2h}V^2\right)},$$

with $C_1=\dfrac{2^8\pi^5 m\varepsilon^{10}}{3\sqrt{3}\,ch^7}, \qquad C_2=\dfrac{2^7\pi^4\varepsilon^{10}}{3\sqrt{3}\,cm^3h^4}.$

The following expressions have been computed by the present author.

$$\frac{1}{N_e N_{010}}\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{ff}}=2\left[\int_V^{\infty}(f)_V\cdot\frac{V'\sigma(\chi'\chi)}{m(V'^2-V^2)}\,\frac{4\pi V'}{m}mV'dV'\right.$$
$$\left.-\int_0^{\infty}(f)_V\cdot\frac{V\sigma(\chi'\chi)}{m(V^2-V'^2)}\,\frac{4\pi V}{m}mV'dV'\right]\frac{m}{4\pi V},$$

where V and V' denote the velocities of free electron before and after the encounter, respectively, and[7] $\sigma(\chi\chi')=\dfrac{2^5\pi^2\varepsilon^6}{3\sqrt{3}\,m^2c^3hV^2}$. Further

$$\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{in. coll}}=\begin{cases}-N_eN_A\omega(AB)\dfrac{(f)_V}{V}+N_eN_A\omega(AB)\dfrac{(f)_{V''}}{V''}\,\dfrac{du''dv''dw''}{dudvdw}, & \\ \qquad\qquad\text{for } \frac{1}{2}mV^2\geqq\chi(AB), & \\ N_eN_A\omega(AB)\dfrac{(f)_{V''}}{V''}\,\dfrac{du''dv''dw''}{dudvdw}, & \\ \qquad\qquad\text{for } 0\leqq\frac{1}{2}mV^2<\chi(AB), & \end{cases}$$

for an inelastic collision exciting an atom from A to B; and

$$\left[\frac{\partial}{\partial t}(N_e f)\right]_{\text{sup. coll}}=\begin{cases}-N_eN_B\omega(BA)\dfrac{(f)_V}{V}+N_eN_B\omega(BA)\dfrac{(f)_{V'}}{V'}\,\dfrac{du'dv'dw'}{dudvdw}, & \\ \qquad\qquad\text{for } \frac{1}{2}mV^2\geqq\chi(AB), & \\ -N_eN_B\omega(BA)\dfrac{(f)_V}{V}, \quad \text{for } 0\leqq\frac{1}{2}mV^2<\chi(AB), & \end{cases}$$

for a superelastic collision, where $\frac{1}{2}m(V'^2-V^2)=\chi(AB)=\frac{1}{2}m(V^2-V''^2)$ is the corresponding excitation potential and $\sigma(AB)=\dfrac{\omega(AB)}{V}$ the cross section for electron collision excitation. The quantum-mechanical values

---

[6] Gaunt, Philosophical Transaction, **229** A (1929), 200.

[7] Menzel and Pekeris, Monthly Notices Roy. Astr. Soc., **96** (1935), 77.

of $\omega(AB)$ for the three quantum states $^3P_{0,1,2}$, $^1D_2$, $^1S_0$ of OIII are computed by Hebb and Menzel[8] and the transition probabilities for spontaneous emissions among these states have been calculated by Pasternack[9].

The ionisations and captures of free electrons of the oxygen atoms are shown to be negligible. The collision excitations and de-excitations of hydrogen atoms are supposed also to be negligible. As the effect of the three separate states $^3P_0$, $^3P_1$, $^3P_2$ has been shown to be small, we consider them to be amalgamated to one single state ^3P. Thus the condition for radiative equilibrium (8) takes the form:

$$\beta_2 = -\frac{\mathfrak{N}_1\mathfrak{R}+\mathfrak{N}_2+\mathfrak{K}_1}{\mathfrak{Z}_1\mathfrak{R}+\mathfrak{Z}_2+\mathfrak{K}_2}, \tag{9}$$

with
$$\mathfrak{R} = \left\{\int_{\nu_1}^{\infty}\frac{I(\nu)}{\nu^3}g_\nu d\nu\right\} \div \left\{\int_{\nu_1}^{\infty}\frac{I(\nu)}{h\nu^4}g_\nu d\nu\right\},$$

$$\mathfrak{N}_1 = \frac{2\Lambda_0}{3m\alpha^2}, \qquad \mathfrak{N}_2 = -\left(h\nu_1+\frac{3}{2}m\alpha^2\right)\mathfrak{N}_1 - \frac{2}{3}\sigma_3 + \Lambda_0 + \frac{2}{3}\Lambda_1 - \frac{2}{3}\frac{C''T_\epsilon}{C},$$

$$\mathfrak{Z}_1 = \frac{2}{3m\alpha^2}(2\sigma_3 - 3\Lambda_0 - 2\Lambda_1),$$

$$\mathfrak{Z}_2 = -\left(h\nu_1+\frac{3}{2}m\alpha^2\right)\mathfrak{Z}_1 + \frac{8}{3}\sigma_3 + \frac{4}{3}\sigma_5 x_1 - 3\Lambda_0 - 4\Lambda_1 - \frac{4}{3}\Lambda_2 - \frac{2}{3}\frac{C''T_\epsilon}{C},$$

$$\Lambda_n = \sum_{s=1}^{\infty} s^{-3}x_s^n e^{x_s} Ei_1(x_s), \quad \sigma_n = \sum_{s=1}^{\infty} s^{-n}, \quad (n=0, 1, 2, 3),$$

$$x_s = \frac{h\nu_s}{m\alpha^2}, \qquad Ei_1(x) = \int_{x_1}^{\infty} e^{-z}\cdot\frac{dz}{z},$$

$$C = \frac{\cdot 2^9\pi^5}{3\sqrt{3}\,(2\pi)^{\frac{3}{2}}}\,\frac{\varepsilon^{10}}{m^2c^3h^3}\cdot\frac{m^{\frac{3}{2}}}{k^{3/2}}, \qquad C'' = \frac{2^7\pi^3}{3\sqrt{3}}\,\frac{\varepsilon^6}{c^3(2\pi m)^{\frac{3}{2}}}\,\frac{1}{hk^{\frac{3}{2}}},$$

$$\mathfrak{K}_1 = k_1^{(P)}\frac{N_P}{N_{010}} + k_1^{(D)}\frac{N_D}{N_{010}} + k_1^{(s)}\frac{N_s}{N_{010}},$$

$$k_1^{(P)} = -\frac{m\alpha^2}{3C_2h}\left\{\omega(PS)\frac{2\chi(PS)}{m\alpha^2}e^{-\chi(PS)/m\alpha^2} + \omega(PD)\frac{2\chi(DP)}{m\alpha^2}e^{-\chi(PD)/m\alpha^2}\right\},$$

$$k_1^{(D)} = -\frac{m\alpha^2}{3C_2h}\left\{\omega(DS)\frac{2\chi(DS)}{m\alpha^2}e^{-\chi(DS)/m\alpha^2} - \omega(DP)\frac{2\chi(DP)}{m\alpha^2}\right\},$$

$$k_1^{(S)} = \frac{m\alpha^2}{3C_2h}\left\{\omega(SP)\frac{2\chi(PS)}{m\alpha^2} + \omega(SD)\frac{2\chi(DS)}{m\alpha^2}\right\},$$

and a similar expression for $\mathfrak{K}_2$, where N_P, N_D, N_S denote respectively the numbers per unit volume of the OIII atoms in the ^3P-, $^1D_2$- and $^1S_0$-states, and ν_s the frequency of the s-th series limit of a hydrogen atom, and g_ν is the Gaunt factor.

---

8) Hebb and Menzel, Astrophys. Journ., **92** (1940), 408.
9) Pasternack, Astrophys. Journ., **92** (1940), 129.

The equation for the cyclic transitions for the ${}^1S_0$-state is

$$N_P\omega(PS)\left\{e^{-\chi(PS)/m\alpha^2}+\beta_2\left(\frac{2\chi(PS)}{m\alpha^2}-1\right)e^{-\chi(PS)/m\alpha^2}\right\}$$
$$+N_D\omega(DS)\left\{e^{-\chi(DS)/m\alpha^2}+\beta_2\left(\frac{2\chi(DS)}{m\alpha^2}-1\right)e^{-\chi(DS)/m\alpha^2}\right\}$$
$$-N_S\left\{\omega(SD)(1-\beta_2)+\omega(SP)(1-\beta_2)+(A_{SP}+A_{SD})\sqrt{\frac{\pi}{2}}\frac{\alpha}{N_e}\right\}=0\,,$$

and similar equations for the other states are written down analogously. From these three equations the ratio $N_P:N_D:N_S$ is determined, which should be substituted in the expressions for $\mathfrak{K}_1$ and $\mathfrak{K}_2$. Thus the ratio N_P/N_{010} is only left undetermined at this stage.

§ 4. For several planetary nebulae we have four kinds of observational data available[10], that is, the absolute intensity $I(c2)$ of the Balmer continuum at the series limit, the intensity ratio r of the Balmer continuum limit in the range $\delta\nu$ to $H\beta$, the intensity ratio of the two nebular lines $I(N_1+N_2)/I(4363)$, and the intensity ratio of the nebular lines to $H\beta$, $I(N_1+N_2)/I(H\beta)$. Now

$$I(c2)=N_eN_{010}\frac{hCg_{\nu_2}}{8T_\varepsilon^{\frac{3}{2}}}(1-3\beta_2)\,,$$

$$r=\frac{1}{A_4(T_\varepsilon)+\beta_2B_4(T_\varepsilon)}\cdot\frac{g_{\nu_2}}{8T_\varepsilon^{\frac{3}{2}}}\delta\nu\cdot(1-3\beta_2)\,,$$

$$\frac{I(N_1+N_2)}{I(4363)}=\frac{N_D}{N_S}\cdot\frac{A_{DP}h\nu_{DP}}{A_{SP}h\nu_{SP}}\,,$$

$$\frac{I(N_1+N_2)}{I(H\beta)}=\frac{A_{DP}h\nu_{DP}}{hC}\,\frac{N_D}{N_P}\,\frac{N_P}{N_eN_{010}}\,\frac{1}{A_4(T_\varepsilon)+\beta_2B_4(T_\varepsilon)}\,,$$

where $A_4(T_\varepsilon)$ and $B_4(T_\varepsilon)$ are known functions of T_ε obtained by studying the equations for cyclic transitions among the various quantum states of a hydrogen atom. From these four kinds of data we compute the values of T_ε, N_e, β_2 and the abundance ratio $N(\text{OIII})/N(\text{HII})$. The result of computation is shown in the first five columns of the table. Under the assumption of the Maxwellian velocity distribution these data can not possibly be brought in accordance with each other.

Nebula	T_ε	N_e	β_2	$\frac{N(\text{OIII})}{N(\text{HII})}$	T_s H & OIII	T_s H	T_s Berman
NGC 6543	5180^0	$1.664\cdot10^4$	+0.227	$8.31\cdot10^{-5}$	32800^0	9170	34150^0
NGC 6826	6640	$0.658\cdot10^4$	+0.223	$3.36\cdot10^{-5}$	32700	11700	27500
NGC 6572	8600	$1.557\cdot10^4$	+0.215	$1.92\cdot10^{-5}$	38150	14440	42500
NGC 7009	8580	$2.456\cdot10^4$	+0.162	$2.76\cdot10^{-5}$	30390	13060	39750
NGC 7662	9960	$1.905\cdot10^4$	+0.217	$2.01\cdot10^{-5}$	60800	16160	47000

---

10) Menzel and Aller, Astrophys. J. **93** (1941), 198; Menzel, Aller and Hebb, *ibid.*, **93** (1941), 230; Aller, *ibid.*, **93** (1941), 236.

We now suppose that the intensity $I(\nu)$ of the incident radiation is that of diluted black body radiation corresponding to the temperature T_s. With the computed values of T_ε, N_e, β_2, $N(\mathrm{OIII})/N(\mathrm{HII})$ we calculate the value of T_s by the condition for radiative equilibrium (9). The result is shown in the sixth column of the table. If we neglect the presence of OIII and compute T_s by the condition for radiative equilibrium in which the terms $\mathfrak{K}_1$ and $\mathfrak{K}_2$ are omitted, then the values come out to be those shown in the seventh column of the table. The effect of mixing OIII atoms is to increase the value of T_s for a given value of T_ε, or conversely, is to lower the value of T_ε for a given value of T_s. The last column of the table shows the values of T_s determined by Berman[11] according to the method of Zanstra[12] from observations of the planetary nebulae.

Thus it is seen that the inclusion of the mechanisms concerning OIII atoms is to make the values of T_s computed on the basis of the condition for radiative equilibrium approach the observed values of T_s, although such values should be modified, as was pointed out by the present author, by removing the assumptions underlying the method of determination. Hence it is expected that the value of T_s computed on the basis of the condition for radiative equilibrium with the inclusion of all the mechanisms concerning all the atoms present in the nebula can be brought to coincidence with the observed value of T_s. At each step of including the mechanisms concerning one new atom the abundance ratio of the atom to hydrogen, for example, should be determined observationally from the spectrophotometric intensity ratios of the spectral lines due to that atom to $H\beta$, for example. In this manner we can not only obtain more accurate values for T_ε, N_e, β_2 and T_s, but also recognise the relative importance of the various mechanisms actually at work in the nebulae. It is on such a final value of β_2 that we can decide if the electron velocity distribution is non-Maxwellian. However we have a good reason to believe that the mechanisms we have taken into account are by far the most important that are at work actually in the planetary nebulae. Hence we are fairly convinced with the fact that the electron velocity distribution in any celestial rarefied gaseous assembly exposed to diluted high frequency radiation is non-Maxwellian.

---

[11] Berman, Lick Observatory Bulletin, No. 430, 1930.

[12] Zanstra, Zeitschr. Astrophys., **2** (1931), 1; Publ. Dominion Astrophys. Obs., **4** (1931), No. 15.