On an integral-type operator between H^2 space and weighted Bergman spaces

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Abstract

Let $H(\mathbb{B})$ denote the space of all holomorphic functions on the unit ball $\mathbb{B}$ of $\mathbb{C}^n$ and $\Re h(z) = \sum_{j=1}^{n} z_j \frac{\partial h}{\partial z_j}(z)$ the radial derivative of h. Motivated by recent results by S. Li and S. Stević (see [8] and [9]), in this paper we study the boundedness and compactness of the following integral operator

$$L_g f(z) = \int_0^1 \Re f(tz) g(tz) \frac{dt}{t}, \quad z \in \mathbb{B},$$

between the Hardy space H^2 and weighted Bergman spaces.

1 Introduction

Let $\mathbb{B}$ be the open unit ball of $\mathbb{C}^n$ and S be the boundary of $\mathbb{B}$. We denote by $H(\mathbb{B})$ the space of all holomorphic functions in $\mathbb{B}$. Let

$$\Re f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z)$$

stand for the radial derivative of $f \in H(\mathbb{B})$. Let dv be the normalized Lebesgue measure on $\mathbb{B}$, i.e. $v(\mathbb{B}) = 1$, and $dv_\alpha(z) = c_\alpha(1 - |z|)^\alpha dv(z)$, where

$$c_\alpha = \frac{\Gamma(n + \alpha + 1)}{\Gamma(n + 1)\Gamma(\alpha + 1)}.$$

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For any $\zeta \in S$ and $r > 0$, the nonisotropic metric ball $Q_r(\zeta)$ is defined as follows (see, e.g. [32])

$$Q_r(\zeta) = \{z \in \mathbb{B} : |1 - \langle z, \zeta \rangle| < r\}. \tag{1}$$

Let μ be a positive Borel measure on $\mathbb{B}$. For all $\zeta \in S$ and $r > 0$, we call μ the α-Carleson measure if there exists a constant $C > 0$ such that

$$\mu(Q_r(\zeta)) \leq Cr^{\alpha}. \tag{2}$$

From [29], we see that μ is an α-Carleson measure if and only if

$$\sup_{a \in \mathbb{B}} \int_{\mathbb{B}} \left(\frac{1 - |a|^2}{|1 - \langle z, a \rangle|^2} \right)^{\alpha} d\mu(z) < \infty. \tag{3}$$

Let $0 < p < \infty$. The Hardy space $H^p = H^p(\mathbb{B})$ consists of all $f \in H(\mathbb{B})$ such that

$$||f||^p_{H^p} = \sup_{0<r<1} \int_S |f(r\zeta)|^p d\sigma(\zeta) < \infty.$$

It is well known that $f \in H^2$ if and only if (see, e.g. [32])

$$||f||^2_{H^2} \asymp |f(0)|^2 + \int_{\mathbb{B}} |\Re f(z)|^2 (1 - |z|^2) dv(z) < \infty. \tag{4}$$

Let $p \in (0, \infty)$ and $\alpha > -1$. The weighted Bergman space $A^p_\alpha = A^p_\alpha(\mathbb{B})$ is defined to be the space of all $f \in H(\mathbb{B})$ such that

$$\|f\|^p_{A^p_\alpha} = \int_{\mathbb{B}} |f(z)|^p dv_\alpha(z) < \infty.$$

It is well known that $f \in A^p_\alpha$ if and only if (see, e.g. [3])

$$\|f\|^p_{A^p_\alpha} \asymp |f(0)|^p + \int_{\mathbb{B}} |\Re f(z)|^p (1 - |z|^2)^p dv_\alpha(z) < \infty. \tag{5}$$

When $\alpha = 0$, A^p_0 is denoted by A^p, which is the classical Bergman space. See [31, 32] for some basic facts on weighted Bergman spaces.

Suppose that $g \in H(\mathbb{B})$. We consider the integral-type operator L_g as follows

$$L_g f(z) = \int_0^1 \Re f(tz) g(tz) \frac{dt}{t}, \qquad f \in H(\mathbb{B}), \qquad z \in \mathbb{B}.$$

This operator is called the Riemann-Stieltjes operator, which was introduced in [5], and was studied in [1, 2, 5, 6, 7, 8, 9, 10, 11, 14, 15, 17, 23, 25, 27, 33, 34]. See [12, 13, 16, 19] for closely related operators in the case of the unit disk, as well as [20, 21, 22, 24, 26, 28, 30] for another related closely integral-type operator on the unit ball.

In [8, 9], S. Li and S. Stević studied the boundedness and compactness of the operator L_g on Hardy spaces and weighted Bergman spaces respectively. The purpose of this paper is to study the boundedness and compactness of the operator L_g between Hardy spaces and weighted Bergman spaces.

Throughout this paper, C will stand for a positive constant, whose value may differ from one occurrence to the other. The expression $a \asymp b$ means that there is a positive constant C such that $C^{-1}a \leq b \leq Ca$.

2 Main results and proofs

In this section we formulate and prove the main results of this paper. For this purpose, we need some auxiliary results which are incorporated in the following lemmas. The following criterion for compactness follows from standard arguments (see, e.g. Proposition 3.11 of [4] or Lemma 3 of [7]).

Lemma 1. *Assume that $g \in H(\mathbb{B})$, $\alpha > -1$ and $0 < p < \infty$. Then the operator $L_g : H^2 \to A^p_\alpha$ is compact if and only if $L_g : H^2 \to A^p_\alpha$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in H^2 which converges to zero uniformly on compact subsets of $\mathbb{B}$, we have $\|L_g f_k\|_{A^p_\alpha} \to 0$ as $k \to \infty$.*

Similarly to the proof of Lemmas 3 and 4 of [9], we have the following two results. We omit the proofs.

Lemma 2. *Assume that $g \in H(\mathbb{B})$, $\alpha > -1$ and $2 \le p \le \frac{2(n+1+\alpha)}{n}$. Then the following two conditions are equivalent.*

(a)

$$\sup_{z\in\mathbb{B}} |g(z)|(1-|z|^2)^{\frac{n+1+\alpha}{p}-\frac{n}{2}} < \infty; \tag{6}$$

(b)

$$\sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{(n+2)p}{2}} |g(z)|^p (1-|z|^2)^p d\nu_\alpha(z) < \infty. \tag{7}$$

Lemma 3. *Assume that $g \in H(\mathbb{B})$, $\alpha > -1$ and $2 \le p \le \frac{2(n+1+\alpha)}{n}$. Then the following two conditions are equivalent.*

(a)

$$\lim_{|z|\to 1} |g(z)|(1-|z|^2)^{\frac{n+1+\alpha}{p}-\frac{n}{2}} = 0; \tag{8}$$

(b)

$$\lim_{|a|\to 1} \int_{\mathbb{B}} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{(n+2)p}{2}} |g(z)|^p (1-|z|^2)^p d\nu_\alpha(z) = 0. \tag{9}$$

Now we are in a position to state and prove the main results of this paper.

Theorem 1. *Suppose that $g \in H(\mathbb{B})$, $\alpha > -1, 2 \le p \le \frac{2(n+1+\alpha)}{n}$. Then $L_g : H^2 \to A^p_\alpha$ is bounded if and only if (6) holds.*

Proof. It is easy to see that $L_g f(0) = 0$ and

$$\Re[L_g(f)](z) = \Re f(z) g(z).$$

By (5), we have

$$\begin{aligned} \|L_g f\|^p_{A^p_\alpha} &\asymp \int_{\mathbb{B}} |\Re(L_g f)(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ &= \int_{\mathbb{B}} |g(z)|^p |\Re f(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ &= \int_{\mathbb{B}} |\Re f(z)|^p d\mu_1(z), \end{aligned} \tag{10}$$

where

$$d\mu_1(z) = |g(z)|^p (1-|z|^2)^p d\nu_\alpha(z). \tag{11}$$

By using the result of [31], we see that $L_g : H^2 \to A^p_\alpha$ is bounded if and only if

$$\mu_1(Q_r(\zeta)) \le C r^{\frac{(n+2)p}{2}}.$$

From this and (3), we have that $L_g : H^2 \to A^p_\alpha$ is bounded if and only if

$$\sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{(n+2)p}{2}} |g(z)|^p(1-|z|^2)^p d\nu_\alpha(z) < \infty.$$

The desired result follows from Lemma 2. The proof is completed.

Theorem 2. *Suppose that $g \in H(\mathbb{B})$, $\alpha > -1, 2 \le p \le \frac{2(n+1+\alpha)}{n}$. Then $L_g : H^2 \to A^p_\alpha$ is compact if and only if (8) holds.*

Proof. Suppose that $L_g : H^2 \to A^p_\alpha$ is compact. Assume that $(a_k)_{k\in\mathbb{N}}$ is a sequence in $\mathbb{B}$ such that $\lim_{k\to\infty} |a_k| = 1$. Set

$$h_k(z) = (1-|a_k|^2)^{\frac{n+2}{2}} \int_0^1 \left(\frac{1}{(1-\langle tz,a_k\rangle)^{(n+2)}} - 1\right)\frac{dt}{t}. \tag{12}$$

By using (4), the fact that $h_k(0) = 0$, and Theorem 1.12 of [32], we obtain

$$\begin{aligned} \|h_k\|^2_{H^2} &\asymp \int_{\mathbb{B}} |\Re h_k(z)|^2 (1-|z|^2) d\nu(z) \\ &\asymp \int_{\mathbb{B}} \frac{(1-|a_k|^2)^{n+2}}{|1-\langle z,a_k\rangle|^{2(n+2)}} (1-|z|^2) d\nu(z) \le C. \end{aligned} \tag{13}$$

Moreover, it is clear that $h_k \to 0$ uniformly on compact subsets of $\mathbb{B}$. Therefore, by Lemma 1 we have that $\|L_g h_k\|_{A^p_\alpha} \to 0$ as $k \to \infty$. Hence

$$\begin{aligned} &\lim_{k\to\infty} \int_{\mathbb{B}} \left(\frac{1-|a_k|^2}{|1-\langle z,a_k\rangle|^2}\right)^{\frac{(n+2)p}{2}} |g(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ =\ &\lim_{k\to\infty} \int_{\mathbb{B}} |\Re(L_g h_k)(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ \asymp\ &\lim_{k\to\infty} \|L_g h_k\|^p_{A^p_\alpha} = 0. \end{aligned} \tag{14}$$

From (14) we see that (9) holds. Then the result follows from Lemma 3.

Conversely, assume that (8) holds, that is, (9) holds. Then for any fixed $\varepsilon > 0$, there exists $\eta_0 \in (0,1)$ such that

$$\int_{\mathbb{B}} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{p(n+2)}{2}} d\mu_1(z) < \varepsilon \tag{15}$$

for all $a \in \mathbb{B}$ with $\eta_0 < |a| < 1$, where μ_1 is defined in (11). Let $r_0 = 1 - \eta_0$. For $\zeta \in S$ and $r \in (0, r_0)$, let $a = (1-r)\zeta$. Then $a \in \mathbb{B}$, $\eta_0 < |a| < 1$,

$$|1 - \langle z, a\rangle| < 2r \quad \text{and} \quad 1 - |a|^2 \geq r,$$

for each $z \in Q_r(\zeta)$. Hence

$$\left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{p(n+2)}{2}} \geq \left(\frac{r}{(2r)^2}\right)^{\frac{p(n+2)}{2}} = \frac{1}{(4r)^{\frac{p(n+2)}{2}}} \tag{16}$$

for each $z \in Q_r(\zeta)$. From (15) and (16), we obtain

$$\begin{aligned} \frac{\mu_1(Q_r(\zeta))}{4^{\frac{p(n+2)}{2}} r^{\frac{p(n+2)}{2}}} &\leq \int_{Q_r(\zeta)} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{p(n+2)}{2}} d\mu_1(z) \\ &\leq \int_{\mathbb{B}} \left(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}\right)^{\frac{p(n+2)}{2}} d\mu_1(z) \\ &< \varepsilon \end{aligned}$$

for all $r \in (0, r_0)$ and $\zeta \in S$. Let $\varepsilon > 0$ be fixed and $\widetilde{\mu_1} \equiv \mu_1 \mid_{\mathbb{B}\setminus(1-r_0)\overline{\mathbb{B}}}$. As in the proof of [9] or [18, Theorem 1.1], we obtain that there exists a constant $C > 0$ such that

$$\widetilde{\mu_1}(Q_r(\zeta)) \leq C\varepsilon r^{\frac{p(n+2)}{2}}, \tag{17}$$

for every $r > 0$. Suppose that $(f_k)_{k\in\mathbb{N}}$ is a sequence in H^2 which converges to 0 uniformly on compact subsets of $\mathbb{B}$ and satisfies $\sup_{k\in\mathbb{N}} \|f_k\|_{H^2} \leq L$. We have

$$\begin{aligned} \|L_g f_k\|^p_{A^p_\alpha} &\asymp \int_{\mathbb{B}} |\Re g(z)|^p |f_k(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ &= \int_{\mathbb{B}} |f_k(z)|^p d\widetilde{\mu_1}(z) + \int_{(1-r_0)\overline{\mathbb{B}}} |f_k(z)|^p d\mu_1(z). \end{aligned} \tag{18}$$

By (17) and using the method of Theorem 1.1 of [18], we see that there exists a positive constant C such that

$$\int_{\mathbb{B}} |f_k(z)|^p d\widetilde{\mu_1} \leq C\varepsilon \|f_k\|^p_{H^2} \leq CL^q\varepsilon, \tag{19}$$

for each $k \in \mathbb{N}$. Moreover, $f_k \to 0$ uniformly on $(1-\delta_0)\overline{\mathbb{B}}$, which implies that the second term in (18) can be made small enough for sufficiently large k. From this and since μ_1 is finite, it follows that

$$\lim_{k\to\infty} \int_{(1-\delta_0)\overline{\mathbb{B}}} |f_k(z)|^p d\mu_1(z) = 0. \tag{20}$$

From (18), (19) and (20), we get that

$$\lim_{k\to\infty} \|L_g f_k\|_{A^p_\alpha} = 0.$$

Employing Lemma 1, the result follows.

Theorem 3. *Suppose that $g \in H(\mathbb{B})$, $0 < p < 2$, $\alpha > -1$. Then the following statements are equivalent.*

(a) $L_g : H^2 \to A^p_\alpha$ is bounded;

(b) $L_g : H^2 \to A^p_\alpha$ is compact;

(c) $g \in A^{\frac{2p}{2-p}}_{\frac{p+2\alpha}{2-p}}$.

Proof. From the proof of Theorem 1 we know that

$$\|L_g f\|^p_{A^p_\alpha} \asymp \int_{\mathbb{B}} |\Re f(z)|^p d\mu_1(z),$$

where $d\mu_1$ is defined by (11). By Theorem 54 of [31], we know that (a) and (b) are equivalent and both are equivalent to the following condition

$$\int_{E(a,r)} \frac{(1-|a|^2)^{n+2}}{|1-\langle z,a\rangle|^{2(n+2)}} d\mu_1(z) \in L^{2/(2-p)}(\nu_1),$$

which is the same as

$$\int_{E(a,r)} |g(z)|^p (1-|z|^2)^p \frac{(1-|a|^2)^{n+2}}{|1-\langle z,a\rangle|^{2(n+2)}} d\nu_\alpha(z) \in L^{2/(2-p)}(\nu_1), \tag{21}$$

where

$$E(z,r) = \{w \in \mathbb{B} : \beta(z,w) < r\}$$

and $\beta(z,w)$ is the distance between z and w in the Bergman metric of $\mathbb{B}$. By the subharmonicity of $|g|^p$, using Lemma 2.24 of [32], we have

$$\begin{aligned} &\int_{E(a,r)} |g(z)|^p (1-|z|^2)^p \frac{(1-|a|^2)^{n+2}}{|1-\langle z,a\rangle|^{2(n+2)}} d\nu_\alpha(z) \\ \geq\ & C(1-|a|^2)^{p+\alpha-1}|g(a)|^p. \end{aligned} \tag{22}$$

Therefore (21) implies that

$$(1-|a|^2)^{p+\alpha-1}|g(a)|^p \in L^{2/(2-p)}(\nu_1),$$

which is the same as

$$\int_{\mathbb{B}} |g(a)|^{\frac{2p}{2-p}} (1-|a|^2)^{\frac{p+2\alpha}{2-p}} d\nu < \infty,$$

i.e. $g \in A^{\frac{2p}{2-p}}_{\frac{p+2\alpha}{2-p}}$.

Conversely, if $g \in A^{\frac{2p}{2-p}}_{\frac{p+2\alpha}{2-p}}$, then by Hölder's inequality, we get

$$\begin{aligned} &\|L_g f\|^p_{A^p_\alpha} \\ \asymp\ & \int_{\mathbb{B}} |\Re f(z)|^p |g(z)|^p (1-|z|^2)^p d\nu_\alpha(z) \\ \leq\ & \left(\int_{\mathbb{B}} |g(z)|^{\frac{2p}{2-p}} (1-|z|^2)^{\frac{p+2\alpha}{2-p}} d\nu(z) \right)^{1-\frac{p}{2}} \left(\int_{\mathbb{B}} |\Re f(z)|^2 (1-|z|^2) d\nu(z) \right)^{\frac{p}{2}}. \end{aligned}$$

From this, and by using the following well-known inequality

$$\int_{\mathbb{B}} |\Re f(z)|^2 (1-|z|^2) d\nu(z) \leq C\|f\|_{H^2}^2,$$

it follows that

$$\|L_g f\|_{A_\alpha^p}^p \leq C\|f\|_{H^2}^p \|g\|^p_{A^{\frac{2p}{2-p}}_{\frac{p+2\alpha}{2-p}}},$$

which means that the operator $L_g : H^2 \to A_\alpha^p$ is bounded. The proof of the theorem is completed.

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