J. Math. Soc. Japan
Vol. 37, No. 1, 1985

The mod 2 semicharacteristic and groups acting freely on manifolds

To the memory of Dr. Takehiko Miyata

By Minoru NAKAOKA

(Received March 1, 1984)

Introduction.

In [8, 9] the author established the equivariant point theorem for a continuous map between manifolds with free involution, and applied it to re-prove the theorems of Milnor [6], Lee [4] and Stong [11] on groups acting freely on manifolds. In this paper, we shall show that the following theorems on group actions are also proved easily by making use of the equivariant point theorem.

THEOREM 1. *Suppose that a group G acts freely on a closed $(2n+1)$-dimensional manifold M. Let $\sigma, \tau \in G$ be elements such that $\tau^2=1$ and $\sigma\tau \neq \tau\sigma$. Then the trace of*

$$(\sigma\tau\sigma^{-1}\tau^{-1})^* : \bigoplus_{k \leq n} H^k(M; \boldsymbol{Z}_2) \longrightarrow \bigoplus_{k \leq n} H^k(M, \boldsymbol{Z}_2)$$

is zero.

This is a generalization of Theorem 1 in Montgomery-Yang [7]; they deal with the case G is a dihedral group.

We denote by $k_2(M)$ the mod 2 semicharacteristic of a closed $(2n+1)$-dimensional manifold M:

$$k_2(M) = \sum_{k \leq n} \dim H^k(M; \boldsymbol{Z}_2) \qquad \text{mod } 2.$$

THEOREM 2. *If a 2-group G acts freely on a closed orientable $(4n+1)$-dimensional manifold M with $k_2(M) \neq 0$, then G is cyclic.*

We denote by $k_0(M)$ the rational semicharacteristic of a closed orientable $(2n+1)$-dimensional manifold, i. e.

$$k_0(M) = \sum_{k \leq n} \dim H^k(M; \boldsymbol{Q}) \qquad \text{mod } 2.$$

If n is even and M admits a free action of $\boldsymbol{Z}_2$, then we have $k_0(M) = k_2(M)$ by the formula of Lusztig-Milnor-Peterson [5]. Therefore Theorem 2 coincides with

---

This research was partially supported by Grant-in-Aid for Scientific Research (No. 59460001), Ministry of Education.

Theorem (5.2) of Becker-Schultz [1], which was proved by regarding $k_0(M)$ as the obstruction to the existence of two linearly independent vector fields on M.

THEOREM 3. *Let M be a closed orientable $(4n+1)$-dimensional manifold with $k_2(M)\neq 0$. Suppose that a finite group G acts freely on M, with trivial action on $H^*(M;\boldsymbol{Z}_2)$. Then G is the direct product of a cyclic 2-group and a group of odd order.*

This is a modification of Theorem D in Davis [3]; he and Weinberger proved Theorem 3 replacing $k_2(M)$, $H^*(M;\boldsymbol{Z}_2)$ with $k_0(M)$, $H^*(M;\boldsymbol{Q})$ respectively. To prove it, they used the surgery semicharacteristic.

It will be understood in this paper that our actions are topological actions on topological manifolds.

§1. Equivariant point theorem.

In this section we shall recall briefly the equivariant point theorem which is used later. See [9] for details.

Let M be a closed m-dimensional topological manifold on which $\boldsymbol{Z}_2$ acts freely, and let τ be the generator of $\boldsymbol{Z}_2$. Then it follows that there exists a symplectic basis $\{v_1, \cdots, v_r, v_1', \cdots, v_r'\}$ of $H^*(M;\boldsymbol{Z}_2)$, i.e. a homogeneous basis such that

$$\langle v_i \smile \tau^* v_j, [M]\rangle=0\,, \quad \langle v_i' \smile \tau^* v_j', [M]\rangle=0\,, \quad \langle v_i \smile \tau^* v_j', [M]\rangle=\delta_{ij}$$

for any i, j. For a continuous map $f: M\to M$, we define a mod 2 integer $\hat{L}(f)$ by

$$\hat{L}(f)=\sum_{i=1}^{r}\langle f^* v_i \smile \tau^* f^* v_i', [M]\rangle\,.$$

This is independent of the choice of symplectic bases of $H^*(M;\boldsymbol{Z}_2)$, and is called the *equivariant Lefschetz number* of f. If $f^*=\mathrm{id}: H^*(M;\boldsymbol{Z}_2)\to H^*(M;\boldsymbol{Z}_2)$ then $\hat{L}(f)=k_2(M)$.

Regard $M\times M$ as a $\boldsymbol{Z}_2$-manifold by permutation of factors. Then an equivariant map $\Delta: M\to M\times M$ is defined by $\Delta(x)=(x, \tau(x))$ $(x\in M)$. Consider the homomorphisms

$$H^0_{\boldsymbol{Z}_2}(M;\boldsymbol{Z}_2)\xrightarrow{\Delta_!} H^m_{\boldsymbol{Z}_2}(M\times M;\boldsymbol{Z}_2)\xrightarrow{\hat{f}^*} H^m_{\boldsymbol{Z}_2}(M;\boldsymbol{Z}_2)\,,$$

where $\Delta_!$ is the Gysin homomorphism and $\hat{f}=(f\times f)\circ\Delta$. Identifying $H^m_{\boldsymbol{Z}_2}(M;\boldsymbol{Z}_2)$ with $H^m(M/\boldsymbol{Z}_2;\boldsymbol{Z}_2)$, we define a mod 2 integer $\hat{I}(f)$ by

$$\hat{I}(f)=\langle \hat{f}^*\Delta_!(1), [M/\boldsymbol{Z}_2]\rangle\,,$$

and call it the *equivariant point index* of f.

Now the *equivariant point theorem* is stated as follows:

THEOREM. (1) *If* $\hat{I}(f)\neq 0$, *then there exists an equivariant point* x *of* f, *i.e.* $x\in M$ *such that* $\tau f(x)=f\tau(x)$. (2) $\hat{I}(f)=\hat{L}(f)$.

§ 2. Applications of the equivariant point theorem.

We shall first give

PROOF OF THEOREM 1. We regard M as a $\boldsymbol{Z}_2$-manifold with the free involution $\tau: M\to M$. Then the equivariant Lefschetz number $\hat{L}(\sigma)$ of $\sigma: M\to M$ is zero.

In fact, if $\hat{L}(\sigma)\neq 0$ then it follows from the equivariant point theorem that there exists a point $x\in M$ such that $\sigma\tau(x)=\tau\sigma(x)$, and hence $\sigma\tau=\tau\sigma$ because the action of G is free.

Take a symplectic basis $\{v_1, \cdots, v_r, v_1', \cdots v_r'\}$ of $H^*(M;\boldsymbol{Z}_2)$ such that $\deg v_i\leqq n$ $(1\leqq i\leqq r)$. Then we can put

$$(\sigma\tau\sigma^{-1}\tau^{-1})^*v_i=\sum_{j=1}^{r}a_{ij}v_j \qquad (a_{ij}\in\boldsymbol{Z}_2),$$

and the trace of

$$(\sigma\tau\sigma^{-1}\tau^{-1})^*:\bigoplus_{k\leq n}H^k(M;\boldsymbol{Z}_2)\longrightarrow\bigoplus_{k\leq n}H^k(M;\boldsymbol{Z}_2)$$

is $\sum_{i=1}^{r}a_{ii}$. On the other hand, it follows that

$$\begin{aligned}\hat{L}(\sigma)&=\sum_{i=1}^{r}\langle\sigma^*v_i\smile\tau^*\sigma^*v_i', [M]\rangle\\&=\sum_{i=1}^{r}\langle\tau^*\sigma^*(\sigma^{-1*}\tau^*\sigma^*v_i\smile v_i'), [M]\rangle\\&=\sum_{i=1}^{r}\langle\sigma^{-1*}\tau^*\sigma^*v_i\smile v_i', (\sigma\tau)_*[M]\rangle\\&=\sum_{i=1}^{r}\langle\tau^*(\sigma\tau\sigma^{-1}\tau^{-1})^*v_i\smile v_i', [M]\rangle\\&=\sum_{i=1}^{r}\sum_{j=1}^{r}a_{ij}\langle\tau^*v_j\smile v_i', [M]\rangle\\&=\sum_{i=1}^{r}a_{ii}\,.\end{aligned}$$

This completes the proof.

The following corollary is immediate.

COROLLARY 1. *Let* M *be a closed odd-dimensional manifold with* $k_2(M)\neq 0$. *If a group* G *acts freely on* M, *with trivial action on* $H^*(M;\boldsymbol{Z}_2)$, *then any element of order* 2 *is in the center of* G.

We have also the following corollary which is a slight generalization of Theorem 1 in [7].

COROLLARY 2. *Suppose that a dihedral group D_{2q} of order $2q$ acts freely on a closed $(2n+1)$-dimensional manifold M. If $\rho\in D_{2q}$ is of order >2, then the trace of*

$$\rho^* : \bigoplus_{k\leqq n} H^k(M;\boldsymbol{Z}_2) \longrightarrow \bigoplus_{k\leqq n} H^k(M;\boldsymbol{Z}_2)$$

is zero.

PROOF. Take a presentation $(\sigma, \tau \mid \sigma^q=\tau^2=(\sigma\tau)^2=1)$ of D_{2q}. Then $\rho=\sigma^i$ with $0<i<q$, and

$$\rho\tau\rho^{-1}\tau^{-1}=\sigma^i\tau\sigma^{-i}\tau=\sigma^{2i}=\rho^2\neq 1 .$$

Therefore $\mathrm{Tr}(\rho^{2*})=0$ by Theorem 1. Since $\mathrm{Tr}(\rho^{*2})=\mathrm{Tr}(\rho^*)$ we have $\mathrm{Tr}(\rho^*)=0$.

We shall next re-prove the following theorem of Stong [**11**]. (See also [**8**], [**9**].)

THEOREM 4. *If $\boldsymbol{Z}_2\times\boldsymbol{Z}_2$ acts freely on a closed odd-dimensional manifold M, then $k_2(M)=0$.*

PROOF. Let $\boldsymbol{Z}_2\times\boldsymbol{Z}_2=\langle\sigma, \tau\rangle$. We regard M as a $\boldsymbol{Z}_2$-manifold with τ, and consider the equivariant Lefschetz number $\hat{L}(\sigma)$ and the equivariant point index $\hat{I}(\sigma)$ of $\sigma : M\to M$. Then it is shown as follows that

$$\text{(i)}\quad \hat{L}(\sigma)=k_2(M) , \qquad\qquad \text{(ii)}\quad \hat{I}(\sigma)=0 .$$

Therefore the equivariant point theorem implies $k_2(M)=0$.

Proof of (i). Take a symplectic basis $\{v_1, \cdots, v_r, v_1', \cdots, v_r'\}$ of $H^*(M;\boldsymbol{Z}_2)$. Then it follows that

$$r=k_2(M)$$

and

$$\begin{aligned}\hat{L}(\sigma)&=\sum_{i=1}^{r}\langle\sigma^*v_i\smile\tau^*\sigma^*v_i', [M]\rangle\\ &=\sum_{i=1}^{r}\langle\sigma^*(v_i\smile\tau^*v_i'), [M]\rangle\\ &=\sum_{i=1}^{r}\langle v_i\smile\tau^*v_i', \sigma_*[M]\rangle\\ &=\sum_{i=1}^{r}\langle v_i\smile\tau^*v_i', [M]\rangle\\ &=r .\end{aligned}$$

Proof of (ii). Put $M'=M/\langle\sigma\rangle$, and let $\pi : M\to M'$ denote the projection. Then $\tau : M\to M$ induces a free involution $\tau' : M'\to M'$. Similarly to M, we regard M' as a $\boldsymbol{Z}_2$-manifold with τ', and regard $M'\times M'$ as a $\boldsymbol{Z}_2$-manifold by permutation of factors. Then $\varDelta : M\to M$ induces an equivariant map $\varDelta' : M'\to M'\times M'$, and we have a commutative diagram

$$\begin{tikzcd} H^0_{Z_2}(M;\boldsymbol{Z}_2) \arrow[r, "\Delta_!"] & H^m_{Z_2}(M\times M;\boldsymbol{Z}_2) \arrow[r, "\hat{\sigma}^*"] & H^m_{Z_2}(M;\boldsymbol{Z}_2) \\ H^0_{Z_2}(M';\boldsymbol{Z}_2) \arrow[u, "\pi^*"] \arrow[r, "\Delta'_!"] & H^m_{Z_2}(M'\times M';\boldsymbol{Z}_2) \arrow[u, "(\pi\times\pi)^*"] \arrow[r, "\Delta'^*"] & H^m_{Z_2}(M';\boldsymbol{Z}_2). \arrow[u, "\pi^*"] \end{tikzcd}$$

(See Lemma 1.5 in [**9**] for the commutativity of the left rectangle.) Thus

$$\begin{aligned} \hat{I}(\sigma) &= \langle \hat{\sigma}^*\Delta_!(1),\ [M/\boldsymbol{Z}_2]\rangle \\ &= \langle \pi^*\Delta'^*\Delta'_!(1),\ [M/\boldsymbol{Z}_2]\rangle \\ &= \langle \Delta'^*\Delta'_!(1),\ \pi_*[M/\boldsymbol{Z}_2]\rangle \\ &= 0. \end{aligned}$$

§ 3. Free actions on manifolds of dimension $4n+1$.

If M is a closed orientable $(4n+1)$-dimensional manifold, then

$$k_2(M)-k_0(M)=\langle W_2(M)\smile W_{4n-1}(M),\ [M]\rangle$$

holds, where $W_i(M)$ is the i-th Stiefel-Whitney class of M([**5**]). If M admits a free action of $\boldsymbol{Z}_2$ then

$$W_i(M)=\pi^*W_i(M/\boldsymbol{Z}_2)$$

holds for the projection $\pi: M\to M/\boldsymbol{Z}_2$. (See Lemma 11.2 in [**9**].) Therefore in this case we have $k_2(M)=k_0(M)$. Using this fact and the results in the preceding section, we shall prove Theorems 2 and 3 in this section.

LEMMA 1. *Let M be a closed orientable $(4n+1)$-dimensional manifold on which $\boldsymbol{Z}_4=(\sigma:\sigma^4=1)$ acts freely. Then we have*

$$k_2(M/\langle\sigma^2\rangle)=k_2(M).$$

PROOF. Consider the isomorphism $\sigma^*: H^k(M;\boldsymbol{C})\to H^k(M;\boldsymbol{C})$ induced by $\sigma: M\to M$. Then we have

$$H^k(M/\langle\sigma^2\rangle;\boldsymbol{C})\cong \mathrm{Ker}(1-\sigma^{*2})$$

where $\boldsymbol{C}$ denotes the complex numbers. (See p. 38 in [**2**].) Since

$$H^k(M;\boldsymbol{C}) \xrightarrow{1-\sigma^{*2}} H^k(M;\boldsymbol{C}) \xrightarrow{1+\sigma^{*2}} H^k(M;\boldsymbol{C})$$

is exact, we have also

$$H^k(M;\boldsymbol{C})\cong \mathrm{Ker}(1-\sigma^{*2})\oplus \mathrm{Ker}(1+\sigma^{*2}).$$

Since the eigenvalues of σ^* are 4th roots of unity, it follows that there exists a basis $\{u_1,\cdots,u_s,v_1,\cdots,v_t\}$ (t: even) of $H^k(M;\boldsymbol{C})$ such that

$$\sigma^{*2}(u_i)=u_i\quad(1\leqq i\leqq s),\qquad \sigma^{*2}(v_j)=-v_j\quad(1\leqq j\leqq t).$$

Therefore $\dim(\mathrm{Ker}(1+\sigma^{*2}))$ is even. Thus it holds

$$\dim H^k(M/\langle\sigma^2\rangle; \boldsymbol{C}) \equiv \dim H^k(M; \boldsymbol{C}) \qquad \mathrm{mod}\ 2\,,$$

which implies

$$k_0(M/\langle\sigma^2\rangle) = k_0(M)\,.$$

A free action of $\boldsymbol{Z}_2$ on $M/\langle\sigma^2\rangle$ is induced by σ, and $M/\langle\sigma^2\rangle$ is orientable, because $\sigma^2: M\to M$ preserves an orientation. Therefore we have

$$k_2(M/\langle\sigma^2\rangle) = k_0(M/\langle\sigma^2\rangle)\,.$$

We have also

$$k_2(M) = k_0(M)\,.$$

Hence $k_2(M/\langle\sigma^2\rangle) = k_2(M)$ holds.

LEMMA 2. *If the quaternion group* $Q(8) = (\alpha, \beta\ ;\ \alpha^2 = \beta^2 = (\alpha\beta)^2)$ *acts freely on a closed orientable* $(4n+1)$*-dimensional manifold* M, *then* $k_2(M)=0$.

PROOF. Since $\boldsymbol{Z}_4 = \langle\alpha\rangle$ acts freely on M, by Lemma 1 we have

$$k_2(M/\langle\alpha^2\rangle) = k_2(M)\,.$$

Since $Q(8)/\langle\alpha^2\rangle \cong \boldsymbol{Z}_2\times\boldsymbol{Z}_2$ acts freely on $M/\langle\alpha^2\rangle$, by Theorem 4 we have

$$k_2(M/\langle\alpha^2\rangle) = 0\,.$$

Therefore $k_2(M)=0$.

PROOF OF THEOREM 2. By Theorem 4, G is a cyclic group or a generalized quaternion group. However the second alternative never occurs by Lemma 2. Thus G is cyclic.

LEMMA 3. *Let* X *be a topological space on which* $\boldsymbol{Z}_2$ *acts freely, and let* $f: X\to X$ *be an equivariant map such that*

$$f^q = \mathrm{id} \qquad \textit{for some odd } q,$$

$$f^* = \mathrm{id} : H^*(X; \boldsymbol{Z}_2) \longrightarrow H^*(X; \boldsymbol{Z}_2)\,.$$

Then we have

$$\bar{f}^* = \mathrm{id} : H^*(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2) \longrightarrow H^*(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)$$

for $\bar{f}: X/\boldsymbol{Z}_2 \to X/\boldsymbol{Z}_2$ *induced by* f.

PROOF. We shall assume inductively $\bar{f}^* = \mathrm{id}: H^k(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)\to H^k(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)$, and prove $\bar{f}^* = \mathrm{id}: H^{k+1}(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)\to H^{k+1}(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)$.

Let $\pi: X\to X/\boldsymbol{Z}_2$ be the projection, and let $w\in H^1(X/\boldsymbol{Z}_2; \boldsymbol{Z}_2)$ be the characteristic class of the 2-fold covering π. Then we have a commutative diagram

$$\begin{array}{ccccc} H^k(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2) & \xrightarrow{\smile w} & H^{k+1}(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2) & \xrightarrow{\pi^*} & H^{k+1}(X;\boldsymbol{Z}_2) \\ \Big\downarrow \bar{f}^*=\mathrm{id} & & \Big\downarrow \bar{f}^* & & \Big\downarrow f^*=\mathrm{id} \\ H^k(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2) & \xrightarrow{\smile w} & H^{k+1}(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2) & \xrightarrow{\pi^*} & H^{k+1}(X;\boldsymbol{Z}_2) \end{array}$$

in which each row is the Gysin exact sequence ([**10**]). Let u be any element of $H^{k+1}(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2)$. Since

$$\pi^*(\bar{f}^*(u)-u)=f^*\pi^*(u)-\pi^*(u)=0\,,$$

there exists $v\in H^k(X/\boldsymbol{Z}_2;\boldsymbol{Z}_2)$ such that

$$\bar{f}^*(u)=u+v\smile w\,.$$

We have then

$$\begin{aligned}\bar{f}^{*2}(u)&=\bar{f}^*(u)+\bar{f}^*(v\smile w)\\ &=u+v\smile w+v\smile w=u\,.\end{aligned}$$

We have also

$$\bar{f}^{*q}(u)=u\,,$$

because $f^q=\mathrm{id}$. Since q is odd we have $\bar{f}^*(u)=u$. This completes the proof.

PROOF OF THEOREM 3. Let G_2 be a 2-Sylow subgroup of G. Then G_2 is cyclic by Theorem 2. We shall prove that G_2 is a normal subgroup of G.

Assume now that G_2 is not normal in G. Let $G_2=\langle\alpha\rangle$. Put

$$l=\mathrm{Min}\{i \mid \langle\alpha^{2^i}\rangle \text{ is normal in } G\}$$

and $\tilde{\alpha}=\alpha^{2^{l-1}}$. Then $l\geqq 1$, and the factor group $G/\langle\tilde{\alpha}^2\rangle$ acts freely on the orbit manifold $M/\langle\tilde{\alpha}^2\rangle$. Therefore Lemma 1 implies that

$$k_2(M/\langle\tilde{\alpha}^2\rangle)=k_2(M/\langle\tilde{\alpha}^4\rangle)=\cdots=k_2(M)\neq 0\,.$$

Let $\beta\in G$ be an element which does not normalize $\langle\tilde{\alpha}\rangle$, and let $\tau,\ \rho\in G/\langle\tilde{\alpha}^2\rangle$ be elements represented by $\tilde{\alpha}$, $\beta\tilde{\alpha}\beta^{-1}$ respectively. It follows that $\tau^2=\rho^2=1$ and that if the order of $\sigma=\rho\tau$ is q then $\langle\tau,\rho\rangle=\langle\sigma,\tau\rangle$ is a dihedral group of order $2q$. In virtue of Theorem 4, q is odd. We see that $\beta\tilde{\alpha}\beta^{-1}\tilde{\alpha}^{-1}$ representing σ is of order 2^iq for some i. Since the order of $(\beta\tilde{\alpha}\beta^{-1}\tilde{\alpha}^{-1})^{2^i}$ is odd q, it follows from Lemma 3 that

$$(\sigma^{2^i})^*=\mathrm{id}\,:\,H^*(M/\langle\tilde{\alpha}^2\rangle;\boldsymbol{Z}_2)\longrightarrow H^*(M/\langle\tilde{\alpha}^2\rangle;\boldsymbol{Z}_2)\,.$$

Therefore, by Corollary 2 of Theorem 1 we have $k_2(M/\langle\tilde{\alpha}^2\rangle)=0$ which contradicts the previous result.

Since the 2-Sylow subgroup G_2 is normal in G, the Schur-Zassenhaus theorem ([**12**]) asserts that G is the semi-direct product of G_2 and a subgroup H of odd order. However, since the group $\mathrm{Aut}(G_2)$ is of even order, the homomorphism $\Phi: H \to \mathrm{Aut}(G_2)$ given by $(\Phi(\theta))(\alpha)=\theta\alpha\theta^{-1}$ $(\theta \in H)$ is trivial. Therefore G is the direct product of G_2 and H. This completes the proof.

References

[1] J. C. Becker and R. E. Schultz, The real semicharacteristic of a fibred manifold, Quart. J. Math. Oxford (2), **33** (1982), 385–403.
[2] A. Borel, Seminar on Transformation Groups, Ann. of Math. Studies, **46**, 1960.
[3] J. F. Davis, The surgery semicharacteristic, Proc. London Math. Soc. (3), **47** (1983), 411–428.
[4] R. Lee, Semicharacteristic classes, Topology, **12** (1973), 183–199.
[5] G. Lusztig, J. Milnor and F. Peterson, Semi-characteristics and cobordism, Topology, **8** (1969), 357–359.
[6] J. Milnor, Groups which act on S^n without fixed points, Amer. J. Math., **79** (1957), 623–630.
[7] D. Montgomery and C. T. Yang, Dihedral group actions I, General Topology and Modern Analysis, Academic Press, 1981, 295–304.
[8] M. Nakaoka, Continuous maps of manifolds with involution I, Osaka J. Math., **11** (1974), 129–145.
[9] M. Nakaoka, Equivariant point theorems for involution, Japan. J. Math., **4** (1978), 263–298.
[10] E. H. Spanier, Algebraic Topology, McGraw-Hill, New York, 1966.
[11] R. Stong, Semi-characteristics and free group actions, Compositio Math., **29** (1974), 223–248.
[12] M. Suzuki, Group Theory I, Springer, Berlin-Heidelberg-New York, 1982.

Minoru NAKAOKA
Department of Mathematics
Faculty of Science
Osaka University
Toyonaka, Osaka 560
Japan