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MAXIMAL IDEALS OF THE CONVOLUTION MEASURE ALGEBRA FOR NONDISCRETE LOCALLY COMPACT ABELIAN GROUPS

Dedicated to Professor Tamotsu Tsuchikura on his sixtieth birthday

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Introduction. Let G be a nondiscrete locally compact abelian group with the dual Γ, and let $M(G)$ be the convolution measure algebra of G (cf. [4]). The algebra $M(G)$ is a commutative Banach algebra with the total variation norm $\| \ \|$, and contains $L^1(G)$ as a closed ideal. As is well-known, the maximal ideal space Δ of $M(G)$ contains Γ, and the restriction to Γ of the Gelfand transform $\hat{\mu}$ of $\mu \in M(G)$ is the Fourier-Stieltjes transform of μ ([6]). A closed subalgebra N (resp. closed ideal) of $M(G)$ is called an L-subalgebra (resp. L-ideal) if a measure ν belongs to N whenever ν is absolutely continuous with respect to a measure belonging to N.

Given a σ-compact set E in G, let $I(E)$ be the set of those measure μ in $M(G)$ which satisfy $|\mu|(Gp(E)+x)=0$ for all $x \in G$, where $Gp(E)$ is the group generated algebraically by E. Let $R(E)$ be the set of those measures in $M(G)$ which are singular with respect to all members of $I(E)$. Thus $I(E)$ and $R(E)$ are an L-ideal and an L-subalgebra of $M(G)$, respectively, and $M(G)$ can be decomposed into the direct sum of $I(E)$ and $R(E)$. Moreover, each measure in $R(E)$ is carried by a countable union of translates of $Gp(E)$. Let P_E denote the natural projection from $M(G)$ onto $R(E)$. Then P_E is multiplicative and the linear functional $\mu \to (P_E\mu)\hat{}\,(1) = (P_E\mu)(G)$ is a nontrivial complex homomorphism of $M(G)$, which we will denote by $h_E \in \Delta$.

Let G_τ be the topological group G with a locally compact group topology τ which is stronger than the original topology of G. Then there exists a σ-compact subset E of G with zero Haar measure such that E is an open subset of G_τ. Dunkl and Ramirez [3] proved that h_E is in the closure $\bar{\Gamma}$ of Γ in Δ. Also Méla [5] and Sato [7] proved that $h_E(\neq 1)$ is in $\bar{\Gamma}$ for some perfect independent subset E. But there do not seem to be many sufficient conditions on a Borel subset E for h_E to be in $\bar{\Gamma}$.

In this paper, we shall investigate h_E for some Borel sets E by developing the ideas in [7]. Then we obtain information on $\Delta\backslash\bar{\Gamma}$.

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1. Preliminaries. For $\mu\in M(G)$, we identify $L^1(\mu)$ with the subspace of absolute continuous measures with respect to μ. An element $\chi=\{\chi_\mu: \mu\in N\}$ of the product space

$$\prod_{\mu\in N} L^\infty(\mu)$$

is called a generalized character of an L-subalgebra N of $M(G)$ if

(a) $\chi_\mu=\chi_\nu$ (ν a.e.) if $\nu\in L^1(\mu)$,

(b) $\chi_{\mu*\nu}(x+y)=\chi_\mu(x)\chi_\nu(y)$ ($\mu\times\nu$ a.e. (x, y)), and

(c) $0<\sup\{\|\chi_\mu\|_\infty: \mu\in N\}\leqq 1$.

Every generalized character χ of N gives rise to a complex homomorphism of N by the formula

$$\mu\to\int\chi_\mu d\mu(=\hat{\mu}(\chi))$$

for every $\mu\in N$. In this way the maximal ideal space $\Delta(N)$ of N can be realized as the set of all generalized characters of N with the topology induced from the $\sigma(L^\infty(\mu), L^1(\mu))$-topology on each factor in the product space (cf. [4]). For $\chi=\{\chi_\mu\}$ and $\xi=\{\xi_\mu\}$ in $\Delta(N)$, we define $\chi\xi$, $\bar{\chi}$ and $|\chi|$ by $(\chi\xi)_\mu=\chi_\mu\xi_\mu$, $(\bar{\chi})_\mu=\bar{\chi}_\mu$ and $|\chi|_\mu=|\chi_\mu|$ for all $\mu\in N$, respectively. For $\mu\in M(G)$, we denote $\|\hat{\mu}\|_\infty=\sup\{|\hat{\mu}(\gamma)|: \gamma\in\Gamma\}$. Given a subset K of G and an integer n, we define $nK=K+\cdots+K$ (n times) if $n>0$, $nK=0$ if $n=0$, and $nK=(-n)(-K)$ if $n<0$, where $-K=\{-x: x\in K\}$. Let K be a subset of G. We say that K is independent (resp. strongly independent) if (a) $0\notin K$, and if (b) whenever $x_1, \cdots, x_n$ are finitely many distinct elements of K, $(p_1, \cdots, p_n)\in \boldsymbol{Z}^n$, and $p_1x_1+\cdots+p_nx_n=0$, then $p_jx_j=0$ (resp. $p_j=0$) for all $j=1, \cdots, n$.

The following characterization for $\bar{\Gamma}$ is useful in this paper.

PROPOSITION 1 ([3]). *Let f be an element in Δ. Then f is contained in $\bar{\Gamma}$ if and only if $|\hat{\mu}(f)|\leqq\|\hat{\mu}\|_\infty$ for all $\mu\in M(G)$.*

2. A sufficient condition for h_K to be in $\bar{\Gamma}$. For a compact subset $E\subset G$ and $\mu\in M(\Gamma)$, we define $\|\hat{\mu}\|_E$ as the supremum norm of $|\hat{\mu}|$ on E.

We can prove the following result by modefying the proof of Theorem 1 in [7].

THEOREM 1. *Let G be a nondiscrete locally compact abelian group, and K a Borel subset such that $K=\bigcup_{n=1}^\infty K_n$, where $\{K_n\}_{n=1}^\infty$ is an increasing sequence of compact subsets of G. Suppose that for any K_n, $\varepsilon>0$ and a compact subset E with $E\cap Gp(K)=\varnothing$, there exists a* *probability measure* $\mu \in M(\Gamma)$ *such that* $\|\hat{\mu} - 1\|_{K_n} < \varepsilon$ *and* $\|\hat{\mu}\|_E < \varepsilon$. *Then we have* $h_K \in \bar{\Gamma}$.

3. Helson set and h_K. In this section, we discuss the relation between a Helson set K and the associated functional h_K.

DEFINITION 1. A compact set K is called an H_α-set $(0 < \alpha \leqq 1)$ if $\alpha = \inf\{\|\hat{\mu}\|_\infty : \|\mu\| = 1,\ \mu \in M(K)\}$. Also we simply call K a Helson set, if K is an H_α-set for some α.

THEOREM 2 ([7]). *Let G be a nondiscrete locally compact abelian group, and K an H_1-set. Then we have* $h_K \in \bar{\Gamma}$.

DEFINITION 2. For a compact set K, we put $S(K) = \{u \in C(K): |u| = 1 \text{ on } K\}$.

Let K be a strongly independent compact subset of G. For $u \in S(K)$, we define a function F_u on $Gp(K)$ by

$$F_u(x) = \prod_{i=1}^{n} u(x_i)^{n_i}$$

for $x = \sum_{i=1}^{n} n_i x_i$ ($n_i \in \boldsymbol{Z}$, all distinct $x_i \in K$).

LEMMA. *Let N be a natural number, and $u \in S(K)$. Then F_u is continuous on $N(K \cup (-K))$.*

PROOF. Let $z = \varepsilon_1 z_1 + \cdots + \varepsilon_N z_N$ be in $N(K \cup (-K))$, where $z_i \in K$ and $\varepsilon_i \in \{\pm 1\}$. Also let $\{z_\alpha\}$ be a net in $N(K \cup (-K))$ such that $z_\alpha = \sum_{i=1}^{N} \varepsilon_{i\alpha} z_{i\alpha}$ ($z_{i\alpha} \in K$, $\varepsilon_{i\alpha} \in \{\pm 1\}$) and $z_\alpha \to z$ as $\alpha \to \infty$. Since K is a compact set, there exists a subset $\{z_{\alpha(\beta)}\}$ of $\{z_\alpha\}$ such that each net $\{z_{i\alpha(\beta)}\}$ coverges to some $y_i \in K$ and each net $\{\varepsilon_{i\alpha(\beta)}\}$ to some $\eta_i \in \{\pm 1\}$. Then we have $z_{\alpha(\beta)} \to \sum_{i=1}^{N} \eta_i y_i$ ($\beta \to \infty$), and $\sum_{i=1}^{N} \eta_i y_i = \sum_{i=1}^{N} \varepsilon_i z_i$. Also $F_u(z_{\alpha(\beta)}) = \prod u(z_{i\alpha(\beta)})^{\varepsilon_{i\alpha(\beta)}} \to \prod u(y_i)^{\eta_i}$ and K is strongly independent. Hence we see that $\prod u(y_i)^{\eta_i} = \prod u(z_i)^{\varepsilon_i}$ and $F_u(z_\alpha) \to F_u(z) (= \prod u(z_i)^{\varepsilon_i}) (\alpha \to \infty)$. q.e.d.

For $u \in S(K)$, we defined a function F_u on $Gp(K)$ by the above rule. By Lemma, F_u is a Borel function on $Gp(K)$, and $|F_u| = 1$ on $Gp(K)$. Then there exists $g \in \Delta$ such that $g_\mu = F_u$ a.e.μ for all $\mu \in M(Gp(K))$ (cf. [4]). We define $\Delta(S(K))$ to be the set of all homomorphisms $g \in \Delta$ associated with some $u \in S(K)$ by the above rule.

DEFINITION 3. Let K be a compact subset. K is called a Kronecker set if for $\varepsilon > 0$ and $u \in S(K)$, there exists $\gamma \in \Gamma$ such that $\|u - \gamma\|_K < \varepsilon$.

It is well-known that a Kronecker set in an H_1-set.

THEOREM 3. *Let K be a strongly independent compact subset of a locally compact abelian group G. If*

$$(*) \qquad \{g \in \Delta(S(K)) \colon |g| = h_K\} \subset \bar{\Gamma} ,$$

then K is an H_1-set. Conversely, if K is a totally disconnected Kronecker set, then $(*)$ holds.

PROOF. The first half is obvious by the definition of an H_1-set.

To prove the second half, suppose K is a totally disconnected Kronecker set, and let Γ_d denote the character group of G_d with discrete topology in G. Furthermore, assume $\chi \in \Gamma_d$, $\chi = 1$ on K, $\{x_j\}_1^n \subset G$, and $\varepsilon > 0$. Then we claim that there exists $\gamma \in \Gamma$ such that $|\chi - \gamma| < \varepsilon$ on $K \cup \{x_1, \cdots, x_n\}$.

Indeed, put $E_\infty = \bigcap_{m=1}^\infty \bar{E}_m$, where

$$(1) \qquad E_m = \{(\gamma(x_1), \cdots, \gamma(x_n)) \mid \gamma \in \Gamma \quad \text{and} \quad \|\gamma - 1\|_K < 1/m\}$$

$(m = 1, 2, \cdots)$. It is easily seen that $E_{m+1} \subset E_m \subset \boldsymbol{T}^n$ for all $m \geqq 1$ and E_∞ forms a compact subgroup of $\boldsymbol{T}^n$. If $(\chi(x_1), \cdots, \chi(x_n)) \notin E_\infty$, it follows that there exists $(p_j) \in \boldsymbol{Z}^n$ such that

$$(2) \qquad p(z) = 1 \quad \text{for all} \quad z \in E_\infty \quad \text{and} \quad \chi(y) \neq 1 ,$$

where $p(z) = p_1^{p_1} \cdots z_n^{p_n}$ for $z = (z_j) \in \boldsymbol{T}^n$ and $y = \sum_{j=1}^n p_j x_j \in G$. By the definition of E_∞ and (2), we can find a natural number m such that $|p(z) - 1| < \varepsilon$ for all $z \in E_m$. This together with (2) shows that $\gamma \in \Gamma$ and $\|\gamma - 1\|_K < 1/m$ imply

$$|\gamma(y) - 1| = \left|\prod_{j=1}^n \gamma(x_j)^{p_j} - 1\right| < \varepsilon .$$

Since $\varepsilon > 0$ is arbitrary, y defines a continuous character of $\{\gamma_{|K} \colon \gamma \in \Gamma\} \subset S(K)$. Since K is a totally disconnected Kronecker set, we infer from Varopoulos' theorem [8] that $y \in Gp(K)$. This contradicts the second condition in (2). Thus $(\chi(x_1), \cdots, \chi(x_n)) \in E_\infty$ and our claim has been confirmed.

Now let $f \in \Delta(S(K))$ be such that $|f| = h_K$ and let μ be a positive measure in $R(K)$. To complete the proof, we may assume that μ is concentrated on $n[K \cup (-K)] + \{x_1, \cdots, x_n\}$ for some finite subset $\{x_j\}$ of G. Set $f(x) = f(\delta_x)$ for all $x \in G$, where δ_x is the unit point-measure at x. Since K is a Kronecker set and $f \in \Delta(S(K))$, there exists a sequence $\{\gamma_m\}$ in Γ such that $\|\gamma_m - f\|_K \mapsto 0$ as $m \mapsto \infty$. Let $\chi \in \Gamma_d$ be any cluster point of $\{\gamma_m\} \subset \Gamma_d$. Then $\bar{\chi} f$ defines an element of Γ_d and $\bar{\chi} f = 1$ on K. It follows from the above paragraph that there exists $\gamma \in \Gamma$ such that $|\gamma - \bar{\chi} f| < \eta/(n+1)$ on $K \cup \{x_1, \cdots, x_n\}$, where $\eta > 0$ is arbitrary. The definition of χ therefore yields an m such that $|\gamma_m \gamma - f| < \eta$ on $n[K \cup (-K)] + \{x_j\}_1^n$. But μ is concentrated on the last set, so

$$\int |\gamma_m \gamma - f| d\mu \leqq \eta \|\mu\| .$$

Since $|f| = h_K \in \bar{\Gamma}$, this completes the proof. q.e.d.

The next result is an improvement of [5; Proposition 12].

PROPOSITION 2. *Let K be an independent compact set. Suppose that $h_E \in \bar{\Gamma}$ for any compact subset E of K with nonempty relative interior. Then K is an H_α-set for some $\alpha(\alpha \geqq 1/8)$.*

PROOF. It is sufficient to show that $\|\mu\| \leqq 8\|\hat{\mu}\|_\infty$ for all $\mu \in M(K)$. First we prove that for a compact subset $E \subset K$ with nonempty relative interior, we have $\left|\int_E d\mu\right| \leqq 2\|\hat{\mu}\|_\infty$ for all $\mu \in M(K)$.

Let μ be in $M(K)$. Since K is independent, we may put $P_E\mu = \mu_1 + \mu_2$, where $\mu_1 \in M(E)$ and μ_2 is a discrete measure with finite support. For $\varepsilon > 0$, there exists $u \in A(G)(=L^1(\Gamma)\hat{\ })$ such that $u = 1$ on $\operatorname{supp} \mu_2$, $u = 0$ on E and $\|u\|_A < 1 + \varepsilon$. Then we have $\left|\int \gamma d\mu_2\right| \leqq \left|\int \gamma u d(P_E u)\right| + \varepsilon$ for all $\gamma \in \Gamma$, and $\|\hat{\mu}_2\|_\infty \leqq \|(P_E\mu)\hat{\ }\|_\infty$. By the assumption that $h_E \in \bar{\Gamma}$, we have $\|(P_E\mu)\hat{\ }\|_\infty \leqq \|\hat{\mu}\|_\infty$, and $\|\hat{\mu}_2\|_\infty \leqq \|\hat{\mu}\|_\infty$. Therefore we obtain

$$\begin{aligned}\left|\int_E \gamma d\mu\right| \leqq \left|\int \gamma d\mu_1\right| &\leqq \left|\int \gamma d\mu\right| + \left|\int \gamma d\mu_2\right| \\ &\leqq \|\hat{\mu}\|_\infty + \|\hat{\mu}\|_\infty , \quad \text{and} \quad \|\hat{\mu}_{|E}\|_\infty \leqq 2\|\hat{\mu}\|_\infty .\end{aligned}$$

Now for $\mu \in M(K)$, we may assume that $\mu = \sum_{m=1}^n \mu_m$, where $\mu_m = \mu_{|I_m}$ and $\{I_m\}_{m=1}^n$ are finitely many disjoint compact subsets of K each with nonempty relative interior. Also there exists a finite set $D \subset \{1, \cdots, n\}$ such that

$$\sum_{m=1}^n |\hat{\mu}_m(1)| \leqq 4 |\sum_{m \in D} \hat{\mu}_m(1)| .$$

We put $A = \bigcup_{m \in D} I_m$. Since by the first half we have $\left|\int_A du\right| \leqq 2\|\hat{\mu}\|_\infty$, we have

$$\sum_{m=1}^n |\hat{\mu}_m(1)| \leqq 4 |\sum_{m \in D} \hat{\mu}_m(1)| \leqq 4\left|\int_A d\mu\right| \leqq 8\|\hat{\mu}\|_\infty .$$

It follows then that $\|\mu\| \leqq 8\|\hat{\mu}\|_\infty$. q.e.d.

4. A maximal ideal depending on a Borel set.

DEFINITION 4. Let E be a compact set of G. E is called a Dirichlet set if $\lim_{\gamma\to\infty} \inf_{\gamma \in \Gamma} \|\gamma - 1\|_E = 0$.

THEOREM 4. *Let G be a nondiscrete locally compact abelian group,* *and K_0 a Dirichlet set in G. Then there exists a σ-compact nonopen subgroup K of G such that $K_0 \subset K$ and $h_K \in \bar{\Gamma}$.*

PROOF. First we notice that if K is as above, then $h_K \notin \Gamma$ since K has Haar measure zero.

Now we choose and fix a σ-compact open subgroup G_0 of G which contains K_0. Since K_0 is a Dirichlet set, there exists a sequence $\{\gamma_k\}$ in Γ such that $\gamma_k \to \infty$ as $k \to \infty$ and such that $\|1 - \gamma_k\|_{K_0} < e^{-k}$ for all $k \geqq 1$. We define

$$(1) \qquad K_n = \bigcap_{k=n^2}^{\infty} \{x \in G_0 : |1 - \gamma_k(x)| \leqq 2e^{-k/n}\} \qquad (n = 1, 2, \cdots) .$$

Then each K_n is σ-compact and has Haar measure zero (notice $\gamma_k \to 1$ uniformly on K_n). Furthermore, we have $-K_n = K_n \subset K_{n+1}$ and $K_n + K_n \subset K_{2n}$ for all n. It follows that $K = \bigcup_{n=1}^{\infty} K_n$ is a σ-compact nonopen subgroup of G containing K_0. We shall prove K fulfills the hypotheses of Theorem 1.

Let E be a compact subset of G which is disjoint from $K = Gp(K)$, and let $n \geqq 2$ be given. Since $E \cap G_0 \cap K_{2n} = \varnothing$, we can find a natural number $N > (2n)^2$ such that

$$(2) \qquad E \cap G_0 \subset \bigcup_{k=n}^{N} 2\{x \in G_0 : |1 - \gamma_k(x)| > 2e^{-k/(2n)}\} .$$

For each integer $k \geqq n^2$, choose an integer m_k so that $n \leqq m_k e^{-k/n} < n + 1$. We set

$$(3) \qquad \psi_n(x) = \prod_{k=n^2}^{N} \left| \frac{1 + \gamma_k(x)}{2} \right|^{2m_k} (x \in G) .$$

Then we claim that ψ_n satisfies

$$(4) \qquad 0 \leqq \psi_n \leqq 1 \quad \text{on} \quad G , \qquad \|\psi_n\|_{B(G)} = 1 ,$$

where $B(G)$ is the set of all Fourier-Stieltjes transforms of $M(\Gamma)$,

$$(5) \qquad \psi_n \leqq e^{-n} \quad \text{on} \quad E \cap G_0 , \quad \text{and}$$

$$(6) \qquad \psi_n \geqq \exp(-6n(n+1)e^{-n}) \quad \text{on} \quad K_n .$$

Part (4) is obvious. To check (5), we pick up any $x \in E \cap G_0$. Then $|1 - \gamma_k(x)| > 2e^{-k/(2n)}$ for some $k \in [n^2, N]$ by (2). Hence

$$\psi_n(x) \leqq \left| \frac{1 + \gamma_k(x)}{2} \right|^{2m_k} = \left[1 - \left| \frac{1 - \gamma_k(x)}{2} \right|^2 \right]^{m_k}$$
$$< (1 - e^{-k/n})^{m_k} < \exp(-m_k e^{-k/n}) < e^{-n}$$

by (3) and our choice of m_k. This establishes (5). If $x \in K_n$, then

$|1-\gamma_k(x)| \leqq 2e^{-k/n}$ for all $k \geqq n^2$ by (1); hence

$$\begin{aligned}\psi_n(x) &= \prod_{k=n^2}^{N}\left[1-\left|\frac{1-\gamma_k(x)}{2}\right|^2\right]^{m_k} \geqq \prod_{k=n^2}^{N}(1-e^{-2k/n})^{m_k}\\ &\geqq \exp\left(-\sum_{k=n^2}^{\infty}(2m_k e^{-2k/n})\right) \geqq \exp\left(-2\sum_{k=n^2}^{\infty}(n+1)e^{k/n}e^{-2k/n}\right)\\ &\geqq \exp\left(-2(n+1)e^{-n}/(1-e^{-1/n})\right),\end{aligned}$$

which establishes (6).

Now let ξ denote the characteristic function of G_0. Since G_0 is an open subgroup of G, we have $\xi \in B(G)$ and $\|\xi\|_{B(G)} = 1$. We define $\phi_n = \psi_n\xi \in B(G)$; then $0 \leqq \phi_n \leqq 1$ on G, $\|\phi_n\|_{B(G)} = 1$ by (4); $\phi_n \leqq e^{-n}$ on E by (5); and $\phi_n = \psi_n \geqq \exp(-6n(n+1)e^{-n})$ on K_n by (6). Since $n \in N$ is arbitrary and $K_n \subset K_{n+1}$ for all n, we conclude that K satisfies the hypotheses of Theorem 1, as desired. q.e.d.

REMARK. Let K be a non H_1-set and perfect strongly independent set. Then there exists $f \in \Delta$ such that $|f| = h_K$, $f \notin \bar{\Gamma}$ and $\hat{\mu}^*(f) = \overline{\hat{\mu}(f)}$ for all $\mu \in M(G)$, where $\mu^*(E) = \overline{\mu(-E)}$.

Indeed, since K is a non H_1-set, there exists $\mu \in M(K)$ such that $\|\mu\| > \|\hat{\mu}\|_\infty$. Then there exists $u \in S(K)$ such that $\left|\int u d\mu\right| > \|\hat{\mu}\|_\infty$. By Lemma, there exists $f \in \Delta(S(K))(|f| = h_K)$ such that $f_\mu(x) = u(x)$ a.e. μ for all $\mu \in M(K)$. So we have $|\hat{\mu}(f)| > \|\hat{\mu}\|_\infty$.

Therefore by Proposition 1, we obtain $f \notin \bar{\Gamma}$. Also f satisfies $\hat{\mu}^*(f) = \overline{\hat{\mu}(f)}$ for all $\mu \in M(G)$. In fact, for $\mu \in R(K)$ we may assume $\mu = \sum_i \mu_{1i} * \mu_{2i}$, where $\{\mu_{1i}\}$ are continuous measures on $Gp(K)$ and $\{\mu_{2i}\}$ are discrete measures on G. Then by the construction of f we have

$$\begin{aligned}\int f d\mu^* &= \sum_i \int f d(\mu_{1i} * \mu_{2i})^*\\ &= \sum_i \int f d\mu_{1i}^* \int f d\mu_{2i}^*\\ &= \sum_i \left(\int f d\mu_{1i}\right)^- \left(\int f d\mu_{2i}\right)^-\\ &= \left(\int f d\mu\right)^-,\end{aligned}$$

where $-$denotes the complex conjugation. q.e.d.

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